CHAPTER 2

Load Flow Analysis

2.1 Introduction

The load flow analysis is a very important and fundamental tool in power system analysis. Its results play the major role during the operational stages of any system for its control and economic schedule, as well as during the expansion and design stages. The purpose of any load flow analysis is to compute precise steady-state voltages and voltage angles of all buses in the network, the real and reactive power flows into every line and transformer, under the assumption of known generation and load. The load flow solution also gives the initial conditions of the system when the transient behaviour of the system is to be studied. In practice it will be required to carry out numerous power flow solutions under a variety of conditions.

2.2 Bus Classification

(i) Load bus: A bus where there is only load connected and no generation exists (both P_{Gi} and Q_{Gi} are zero) is called a load bus. At this bus real power (P_{Di}) and reactive power (Q_{Di}) are drawn from the supply. A load bus is also called a *PQ* bus, since the real power and reactive power are known values at this bus. The other two unknown quantities at a load bus are voltage magnitude ($|V_i|$) and its phase angle (δ_i) at the bus. In a power balance equation P_{Di} and Q_{Di} are treated as negative quantities since generated powers P_{Gi} and Q_{Gi} are assumed positive.

(ii) Voltage controlled bus or generator bus: A voltage controlled bus is any bus in the system where the voltage magnitude can be controlled. At each bus to which there is an alternator connected, the MW generation can be controlled by adjusting the prime mover. In other words, the phase angle of the rotor δ is directly related to the real power generated by the machine. The voltage magnitude can be controlled by adjusting generator excitation. Thus at a generator bus the real power generation (P_{Gi}) and the voltage magnitude ($|V_i|$) can be specified. The phase angle (δ_i) and the reactive power (Q_{Di}) are to be determined. The limits on the value of the reactive power are also specified. These buses are called *PV* buses.

(iii) Slack bus: In a power system network as load flows from the generators to the loads through transmission lines, the power loss occurs due to the losses in the transmission line conductors. These losses when included, we get the power balance relations:

$$P_L = \sum_{i=1}^{N} P_{Gi} - \sum_{i=1}^{N} P_{Di}$$

$$Q_L = \sum_{i=1}^{N} Q_{Gi} - \sum_{i=1}^{N} Q_{Di}$$

where P_{Gi} and Q_{Gi} are the total real and reactive power generations, P_{Di} and Q_{Di} are the total real and reactive power demands and P_L and Q_L are the power losses in the transmission network. The values of P_{Gi}, Q_{Gi}, P_{Di} and Q_{Di} are either known or estimated. For this reason, the slack bus is also known as the reference bus.

Types of Bus	*Specified quantities*	*Quantities to be determined*
Slack or swing or reference bus	$\|V\|$, δ	P, Q
Generator or voltage controlled or *PV* bus	P, $\|V\|$	Q, δ
Load bus or *PQ* bus	P, Q	$\|V\|$, δ

2.3 Load Flow Equation

The relationship between node current and voltage in the linear network can be described by the following node equation:

$$I = YV \tag{2.1}$$

or

$$I_i = \sum_{j=1}^{n} Y_{ij} V_j \qquad i = 1,2,3, \ldots, n \tag{2.2}$$

where I_i and V_j are the injected current at bus i and voltage at bus j, respectively. The voltage at a typical bus i of the system in polar coordinates is given by

$$V_i = |V_i|\angle\delta_i = |V_i| (\cos \delta_i + j \sin \delta_i) \tag{2.3}$$

Y_{ij} an element of the admittance matrix, is given by

$$Y_{ij} = |Y_{ij}|\angle\theta_{ij} = |Y_{ij}| \cos \theta_{ij} + j|Y_{ij}| \sin \theta_{ij} = G_{ij} + jB_{ij} \tag{2.4}$$

where n is the total number of nodes in the system.

The complex power injected by the source into the *i*th bus of a power system is

$$S_i = P_i + jQ_i = V_i I_i^* \qquad i = 1, 2, 3, ..., n \tag{2.5}$$

The complex conjugate of the above equation,

$$P_i - jQ_i = V_i^* I_i \qquad i = 1, 2, 3, ..., n \tag{2.6}$$

We know that

$$I_i = \sum_{j=1}^{n} Y_{ij} V_j$$

Equation (2.6) becomes

$$P_i - jQ_i = V_i^* \sum_{j=1}^{n} Y_{ij} V_j \tag{2.7}$$

Hence basically, real power

$$P_i = \text{real}\left[V_i^* \sum_{j=1}^{n} Y_{ij} V_j \right] \tag{2.8}$$

Reactive power,

$$Q_i = -\,\text{Im}\left[V_i^* \sum_{j=1}^{n} Y_{ij} V_j \right] \tag{2.9}$$

The power flow equations can also be written as follows.
Real power,

$$P_i = |V_i| \sum |V_j||Y_{ij}| \cos(\theta_{ij} + \delta_j - \delta_i) \tag{2.10}$$

Reactive power,

$$Q_i = -|V_i| \sum |V_j||Y_{ij}| \sin(\theta_{ij} + \delta_j - \delta_i) \tag{2.11}$$

Equations (2.10) and (2.11) comprise the polar form of the load flow equations or static load flow equations. They are usually expressed in the following forms as mathematical models of the load flow problem:

$$\Delta P_i = P_{i,\text{sch}} - P_{i,\text{calc}} = (P_{Gi} - P_{Di}) - P_{i,\text{calc}} \tag{2.12}$$

$$\Delta Q_i = Q_{i,\text{sch}} - Q_{i,\text{calc}} = (Q_{Gi} - Q_{Di}) - Q_{i,\text{calc}} \tag{2.13}$$

where $P_{i,\text{sch}}$, $Q_{i,\text{sch}}$ are the specified active and reactive powers at node i based on the above two simultaneous equations. The load flow problem can be roughly summarized as: for specified $P_{i,\text{sch}}$ and $Q_{i,\text{sch}}$, find the voltage vector $|V_i|$ and δ_i such that the magnitudes of the power errors ΔP_i and ΔQ_i are less than the acceptable tolerance.

The functions P_i and Q_i of Eqs. (2.10) and (2.11) are nonlinear functions of the state variables $|V_i|$ and δ_i. This static load flow equations are of such complexity that it is not possible to obtain the exact analytical solution. Hence, the power flow calculations usually employ iterative techniques.

2.4 Load Flow Methods

The iterative techniques are:

1. Gauss–Seidel method
2. Newton–Raphson method
3. Fast decoupled method

2.5 Gauss–Seidel Method

The load flow problem formulated as a set of nonlinear algebraic equations can be solved by an iterative algorithm called the Gauss–Seidel method.

2.5.1 Gauss–Seidel Method When *PV* Buses are Absent

We have chosen the Gauss–Seidel method first because of its simplicity. Now we shall consider the case when the generator buses or voltage controlled buses or *PV* buses are absent. This means we have $n - 1$ load buses or *PQ* buses, the remaining one being the slack bus.

Computational procedure

1. Form the bus admittance matrix of the network by direct inspection method, selecting the ground as reference [formation of Y_{bus}].
2. If the slack bus is not specified, select one of the generator buses as the slack bus. The voltage at the slack bus is assumed as $V_i = V + j0.0$ [selection of the slack bus].
3. Assume initial values of voltages for all buses except the slack bus. $V_i^{(0)} = 1 + j0.0$ (flat start voltage).
4. Set convergence criterion = ε, i.e. if the largest of absolute of the residues exceeds the convergence criterion the process is repeated, otherwise it is terminated.
5. Set iteration count $k = 0$.
6. Bus count $i = 1$. If i is the slack bus, then there will be an increment in the bus count.
7. Solve the voltage equation for bus i as we know that

$$P_i - jQ_i = V_i^* \sum_{j=1}^{n} Y_{ij} V_j$$

$$\frac{P_i - jQ_i}{V_i^*} = V_i \sum_{j=0}^{n} Y_{ij} - \sum_{j=1}^{n} Y_{ij} V_j \quad j \neq i$$

$$V_i = \frac{1}{Y_{ii}} \left[\frac{P_i - jQ_i}{V_i^*} - \sum_{\substack{j=1 \\ j \neq i}}^{n} Y_{ij} V_j \right]$$

or $$V_i = \frac{1}{Y_{ii}}\left[\frac{P_i - jQ_i}{V_i^*} - \sum_{j=1}^{i-1} Y_{ij}V_j - \sum_{j=i+1}^{n} Y_{ij}V_j\right]$$

$$(V_i)^{k+1} = \frac{1}{Y_{ii}}\left[\frac{P_i - jQ_i}{(V_i^k)^*} - \sum_{j=1}^{i-1} Y_{ij}(V_j)^{k+1} - \sum_{j=i+1}^{n} Y_{ij}(V_j)^k\right] \tag{2.14}$$

8. Calculate the change in bus voltage

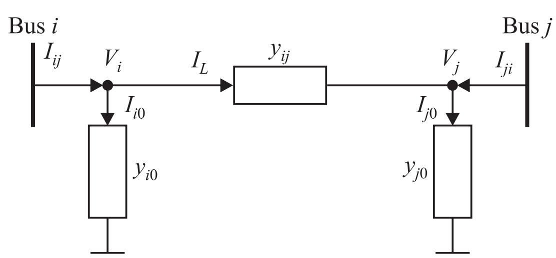
$$\Delta V_i^{k+1} = (V_i)^{k+1} - (V_i)^k \tag{2.15}$$

9. Acceleration of convergence: The process of convergence in the Gauss–Seidel method is slow as it requires larger number of iterations to obtain the solution. In this method, convergence can be increased by using the acceleration factor, denoted by α. In power flow studies, α is generally set about 1.6 and cannot exceed 2 if convergence is to occur. Therefore,

$$V_{i,\text{acc}}^{k+1} = (V_i)^k + \alpha\, \Delta V_i^{k+1} \tag{2.16}$$

Calculate the bus voltages, i.e. V_i^{k+1} for all the buses except the slack bus, where $i = 1, 2, 3, ..., n$.

10. Repeat the iterating process until change in voltage (ΔV_i) for all the buses are within the specified or within the tolerance.
11. Finally calculate the power flow and power losses.



Current and power flows

$$i \rightarrow j$$

$$I_{ij} = I_L + I_{i0} = y_{ij}(V_i - V_j) + y_{i0}V_i$$
$$S_{ij} = V_iI_{ij}^* = V_i^2(y_{ij} + y_{i0})^* - V_iy_{ij}^*V_j^*$$

$$j \rightarrow i$$

$$I_{ji} = -I_L + I_{j0} = y_{ij}(V_j - V_i) + y_{j0}V_j$$
$$S_{ji} = V_jI_{ij}^* = V_j^2(y_{ij} + y_{j0})^* - V_jy_{ij}^*V_i^*$$

Power loss

$$S_{\text{loss}ij} = S_{ij} + S_{ji}$$

This completes the load flow study. Finally, in Figure 2.1 all the computational steps are summarized in the detailed flow chart.

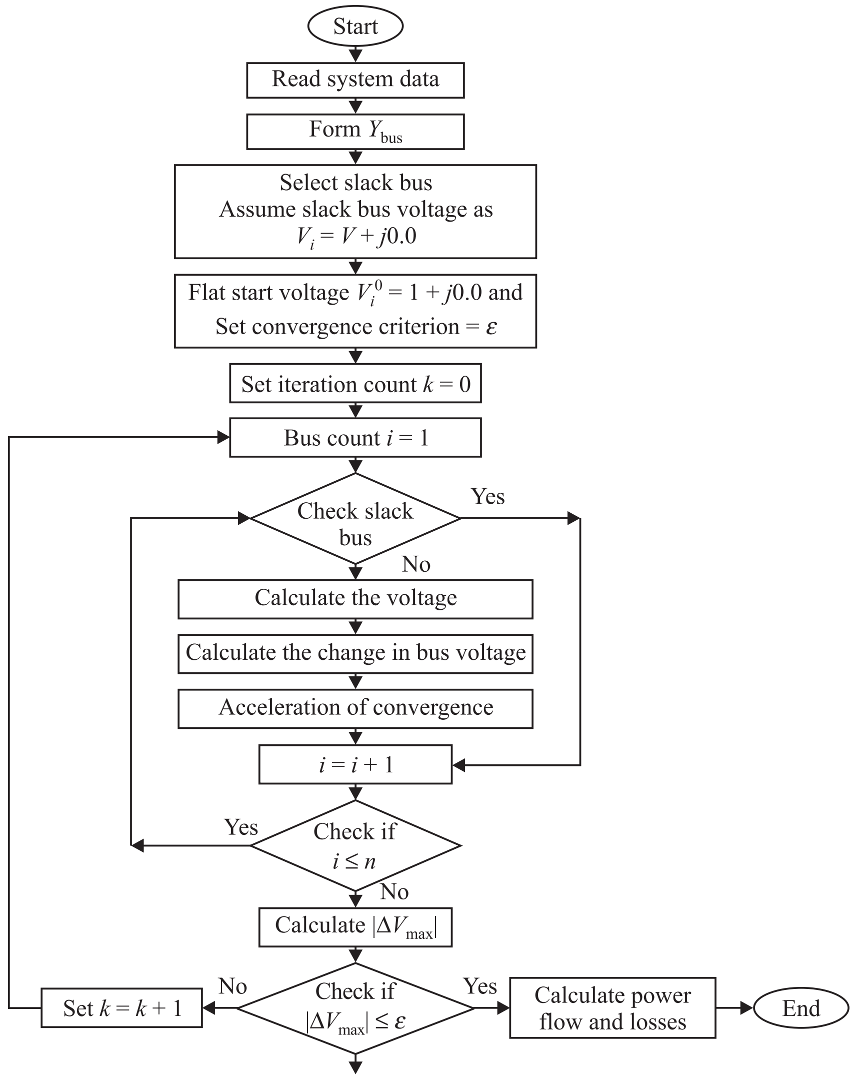


Figure 2.1 Flow chart for Gauss–Seidel method when *PV* buses are absent.

EXAMPLE 2.1 The per unit admittances are indicated at the diagram and the bus data are given in Table 2.1. Determine the voltages at buses 2 and 3 after the first iteration using the Gauss–Seidel method. Assume $\alpha = 1.6$.

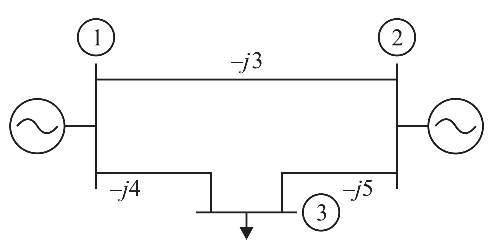


Table 2.1 Bus data

Bus No.	***Bus type***	***Generation (per unit)***		***Load (per unit)***		***Bus voltage***	
		P_G	Q_G	P_D	Q_D	V	δ
1	Slack	—	—	—	—	1.02	0
2	*PQ*	0.25	0.15	0.5	0.25	—	—
3	*PQ*	0	0	0.6	0.3	—	—

Solution: Form the Y_{bus}

$$Y_{11} = y_{12} + y_{13} = -j3 + (-j4) = -j7$$
$$Y_{12} = Y_{21} = -y_{12} = -(-j3) = j3$$
$$Y_{13} = Y_{31} = -y_{13} = -(-j4) = j4$$
$$Y_{22} = y_{21} + y_{23} = -j3 + (-j5) = -j8$$
$$Y_{23} = Y_{32} = -y_{23} = -(-j5) = j5$$
$$Y_{33} = y_{31} + y_{32} = -j4 + (-j5) = -j9$$

$$Y_{\text{bus}} = j\begin{bmatrix} -7 & 3 & 4 \\ 3 & -8 & 5 \\ 4 & 5 & -9 \end{bmatrix}$$

At bus 2, $P_2 = P_{G2} - P_{D2} = 0.25 - 0.5 = -0.25$ p.u.

$Q_2 = Q_{G2} - Q_{D2} = 0.15 - 0.25 = -0.1$ p.u.

At bus 3, $P_3 = P_{G3} - P_{D3} = 0 - 0.6 = -0.6$ p.u.

$Q_3 = Q_{G3} - Q_{D3} = 0 - 0.3 = -0.3$ p.u.

First iteration

Set $k = 0$, bus 1 is the slack bus.

$$\therefore \quad V_1^0 = V_1^1 = V_1^2 = V_1^3 = \cdots = 1.02 + j0.0$$

Assume a flat start voltage for *PQ* buses.

$$V_2^0 = 1\angle 0; V_3^0 = 1\angle 0$$

The voltage at bus 2 is

$$(V_2)^1 = \frac{1}{Y_{22}}\left[\frac{P_2 - jQ_2}{(V_2^0)^*} - Y_{21}V_1 - Y_{23}(V_3)^0\right]$$

$$= \frac{1}{-j8}\left[\frac{-0.25 + j0.1}{1\angle 0} - j3 \times 1.02\angle 0 - j5 \times 1\angle 0\right]$$

$$= 0.995 - j0.03125$$

$$\Delta V_2^1 = (V_2)^1 - (V_2)^0 = (0.995 - j0.03125) - (1 + j0.0) = -0.005 - j0.03125$$

$$V_{2,\text{acc}}^1 = (V_2)^0 + \alpha\,\Delta V_2^1 = (1 + j0.0) + 1.6 \times (-0.005 - j0.03125)$$

$$= 0.992 - j0.0499$$

The voltage at bus 3 is

$$(V_3)^1 = \frac{1}{Y_{33}}\left[\frac{P_3 - jQ_3}{(V_3^0)^*} - Y_{31}V_1 - Y_{32}(V_2)^1\right]$$

$$= \frac{1}{-j9}\left[\frac{-0.6 + j0.3}{1\angle 0} - j4\times 1.02\angle 0 - j5\times(0.992 - j0.0499)\right]$$

$$= 0.971 - j0.0944$$

$$\Delta V_3^1 = (V_3)^1 - (V_3)^0 = (0.971 - j0.0944) - (1 + j0.0) = -0.029 - j0.0944$$

$$V_{3,\text{acc}}^1 = (V_3)^0 + \alpha\,\Delta V_3^1 = (1 + j0.0) + 1.6 \times (-0.029 - j0.0944)$$

$$= 0.9536 - j0.1514$$

The bus voltages at the end of the first iteration are

$$V_1^1 = 1.02 + j0$$
$$V_2^1 = 0.992 - j0.0499$$
$$V_3^1 = 0.9536 - j0.1514$$

EXAMPLE 2.2 The system data for a load flow solution are given in Tables 2.2 and 2.3. Determine the voltages at the end of the first iteration using the Gauss–Seidel method. Take $\alpha = 1.6$.

Table 2.2 Line admittances

Bus code	*Admittance*
1–2	2 – *j*8.0
1–3	1 – *j*4.0
2–3	0.666 – *j*2.664
2–4	1 – *j*4.0
3–4	2 – *j*8.0

Table 2.3 Schedule of active and reactive powers

Bus code	*P in* p.u.	*Q in* p.u.	*V in* p.u.	*Remarks*
1	—	—	1.06	Slack
2	0.5	0.2	1 + *j*0.0	PQ
3	0.4	0.3	1 + *j*0.0	PQ
4	0.3	0.1	1 + *j*0.0	PQ

Solution:

$$Y_{11} = y_{12} + y_{13} = (2 - j8) + (1 - j4) = 3 - j12$$

$$Y_{12} = Y_{21} = -y_{12} = -(2 - j8) = -2 + j8$$

$$Y_{13} = Y_{31} = -y_{13} = -(1 - j4) = -1 + j4$$

$$Y_{22} = y_{21} + y_{23} + y_{24} = (2 - j8) + (0.666 - j2.664) + (1 - j4)$$

$$= 3.666 - j14.664$$

$$Y_{23} = Y_{32} = -y_{23} = -(0.666 - j2.664) = -0.666 + j2.664$$
$$Y_{24} = Y_{42} = -y_{24} = -(1 - j4) = -1 + j4$$
$$Y_{33} = y_{31} + y_{32} + y_{34} = (1 - j4) + (0.666 - j2.664) + (2 - j8)$$
$$= 3.666 - j14.664$$
$$Y_{34} = Y_{43} = -y_{34} = -(2 - j8) = -2 + j8$$
$$Y_{44} = y_{42} + y_{43} = (1 - j4) + (2 - j8) = 3 - j12$$

$$Y_{\text{bus}} = \begin{bmatrix} 3 - j12 & -2 + j8 & -1 + j4 & 0 \\ -2 + j8 & 3.666 - j14.664 & -0.666 + j2.664 & -1 + j4 \\ -1 + j4 & -0.666 + j2.664 & 3.666 - j14.664 & -2 + j8 \\ 0 & -1 + j4 & -2 + j8 & 3 - j12 \end{bmatrix}$$

At bus 2, $P_2 = P_{G2} - P_{D2} = 0 - 0.5 = -0.5$ p.u.

$Q_2 = Q_{G2} - Q_{D2} = 0 - 0.2 = -0.2$ p.u.

At bus 3, $P_3 = P_{G3} - P_{D3} = 0 - 0.4 = -0.4$ p.u.

$Q_3 = Q_{G3} - Q_{D3} = 0 - 0.3 = -0.3$ p.u.

At bus 4, $P_4 = P_{G4} - P_{D4} = 0 - 0.3 = -0.3$ p.u.

$Q_4 = Q_{G4} - Q_{D4} = 0 - 0.1 = -0.1$ p.u.

First iteration

Set $k = 0$, bus 1 is slack bus.

$$\therefore \quad V_1^0 = V_1^1 = V_1^2 = V_1^3 = \cdots = 1.06 + j0.0$$

Assume a flat start voltage for PQ buses

$$V_2^0 = 1\angle 0; V_3^0 = 1\angle 0; V_4^0 = 1\angle 0$$

The voltage at bus 2 is

$$(V_2)^1 = \frac{1}{Y_{22}}\left[\frac{P_2 - jQ_2}{(V_2^0)^*} - Y_{21}V_1^1 - Y_{23}V_3^0 - Y_{24}V_4^0\right]$$

$$V_2^1 = \frac{1}{3.666 - j14.664}\left[\frac{-0.5 + j0.2}{1 - j0.0} - 1.06(-2 + j8)\right.$$
$$\left. -1.0(-0.666 + j2.664) - (-1 + j4)1.0\right]$$
$$= 1.01187 - j0.02888$$

$$\Delta V_2^1 = (V_2)^1 - (V_2)^0 = (1.01187 - j0.02888) - (1 + j0.0)$$
$$= 0.01187 - j0.02888$$

$$V_{2,\text{acc}}^1 = (V_2)^0 + \alpha\,\Delta V_2^1 = (1 + j0.0) + 1.6 \times (0.01187 - j0.02888)$$
$$= 1.01896 - j0.04621$$

The voltage at bus 3 is

$$(V_3)^1 = \frac{1}{Y_{33}}\left[\frac{P_3 - jQ_3}{(V_3^0)^*} - Y_{31}V_1^1 - Y_{32}V_2^1 - Y_{34}V_3^0\right]$$

$$V_3^1 = \frac{1}{3.666 - j14.664}\left[\frac{-0.4 + j0.3}{1 - j0} - (-1 + j4)(1.06) - (-0.666 + j2.664)(1.01187 - j0.02888) - (-2 + j8)(1)\right]$$

$$= 0.9926 - j0.026$$

$$\Delta V_3^1 = (V_3)^1 - (V_3)^0 = (0.9926 - j0.026) - (1 + j0.0)$$

$$= -7.4 \times 10^{-3} - j0.026$$

$$V_{3,\text{acc}}^1 = (V_3)^0 + \alpha\, \Delta V_3^1 = (1 + j0.0) + 1.6 \times (-7.4 \times 10^{-3} - j0.026)$$

$$= 0.988 - j0.0416$$

The voltage at bus 4 is

$$(V_4)^1 = \frac{1}{Y_{44}}\left[\frac{P_4 - jQ_4}{(V_4^0)^*} - Y_{41}V_1^1 - Y_{42}V_2^1 - Y_{43}V_3^1\right]$$

$$V_4^1 = \frac{1}{3 - j12}\left[\frac{-0.3 + j0.1}{1 - j0} - (0)(1.06) - (-1 + j4)(1.01187 - j0.02888) - (-2 + j8)(0.988 - j0.0416)\right]$$

$$= 0.9825 - j0.06$$

$$\Delta V_4^1 = (V_4)^1 - (V_4)^0 = (0.9825 - j0.06) - (1 + j0.0)$$

$$= -0.0175 - j0.06$$

$$V_{4,\text{acc}}^1 = (V_4)^0 + \alpha\, \Delta V_4^1 = (1 + j0.0) + 1.6 \times (-0.0175 - j0.06)$$

$$= 0.9721 - j0.096$$

The bus voltages at the end of the first iteration are

$$V_1^1 = 1.06 + j0$$

$$V_2^1 = 1.01896 - j0.04621$$

$$V_3^1 = 0.988 - j0.0416$$

$$V_4^1 = 0.9721 - j0.096$$

EXAMPLE 2.3 Figure 2.2 shows the one line diagram of a simple three bus system with generation at bus 1. The magnitude of voltage at a bus 1 is adjusted to 1.05 p.u. The scheduled loads at buses 2 and 3 are as marked in the diagram. The line impedances are marked in p.u. on a 100 MVA base and the line charging susceptances are neglected.

(a) Using the Gauss–Seidel method, determine the phasor values of the voltages at the load buses 2 and 3 (*P*-*Q* buses) accurate to decimal places.
(b) Verify the result with Power World Simulator and PSS/E.

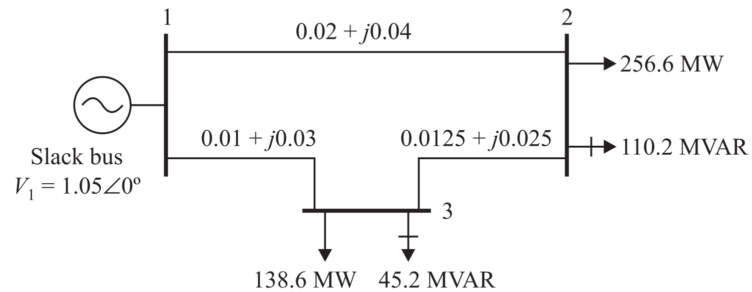


Figure 2.2 One line diagram of a simple three bus system.

Solution: (a) To form the Y_{bus}

$$y_{12} = \frac{1}{z_{12}} = \frac{1}{0.02 + j0.04} = 10 - j20$$

$$y_{13} = \frac{1}{z_{13}} = \frac{1}{0.01 + j0.03} = 10 - j30$$

$$y_{23} = \frac{1}{z_{23}} = \frac{1}{0.0125 + j0.025} = 16 - j32$$

$$Y_{11} = y_{12} + y_{13} = (10 - j20) + (10 - j30) = 20 - j50$$

$$Y_{12} = Y_{21} = -y_{12} = -(10 - j20) = -10 + j20$$

$$Y_{13} = Y_{31} = -y_{13} = -(10 - j30) = -10 + j30$$

$$Y_{22} = y_{21} + y_{23} = (10 - j20) + (16 - j32) = 26 - j52$$

$$Y_{23} = Y_{32} = -y_{23} = -(16 - j32) = -16 + j32$$

$$Y_{33} = y_{31} + y_{32} = (10 - j30) + (16 - j32) = 26 - j62$$

$$Y_{bus} = \begin{bmatrix} 20 - j50 & -10 + j20 & -10 + j30 \\ -10 + j20 & 26 - j52 & -16 + j32 \\ -10 + j30 & -16 + j32 & 26 - j62 \end{bmatrix}$$

At bus 2,
$$P_2 = P_{G2} - P_{D2} = 0 - \frac{256.6}{100} = -2.566 \text{ p.u.}$$

$$Q_2 = Q_{G2} - Q_{D2} = 0 - \frac{110.2}{100} = -1.102 \text{ p.u.}$$

At bus 3,
$$P_3 = P_{G3} - P_{D3} = 0 - \frac{138.6}{100} = -1.386 \text{ p.u.}$$

$$Q_3 = Q_{G3} - Q_{D3} = 0 - \frac{45.2}{100} = -0.452 \text{ p.u.}$$

First iteration

Set $k = 0$, bus 1 is the slack bus.

$$\therefore \quad V_1^0 = V_1^1 = V_1^2 = V_1^3 = \cdots = 1.05 + j0.0$$

Assume a flat start voltage for PQ buses

$$V_2^0 = 1\angle 0;\ V_3^0 = 1\angle 0$$

The voltage at bus 2 is

$$(V_2)^1 = \frac{1}{Y_{22}}\left[\frac{P_2 - jQ_2}{(V_2^0)^*} - Y_{21}V_1^1 - Y_{23}V_3^0 - Y_{24}V_4^0\right]$$

$$V_2^1 = \frac{1}{26 - j52}\left[\frac{-2.566 + j1.102}{1 - j0.0} - (-10 + j20)1.05 - (-16 + j32)1.0\right]$$

$$= 0.9825 - j0.0310$$

The voltage at bus 3 is

$$(V_3)^1 = \frac{1}{Y_{33}}\left[\frac{P_3 - jQ_3}{(V_3^0)^*} - Y_{31}V_1^1 - Y_{32}V_2^1\right]$$

$$V_3^1 = \frac{1}{26 - j62}\left[\frac{-1.386 + j0.452}{1 - j0} - (-10 + j30)(1.05) - (-16 + j32)(0.9825 - j0.0310)\right] = 1.0011 - j0.0353$$

The bus voltages at the end of the first iteration are

$$V_1^1 = 1.05 + j0$$

$$V_2^1 = 0.9825 - j0.0310$$

$$V_3^1 = 1.0011 - j0.0353$$

(b) **Verify the result using Power World Simulator (PWS):** The one line diagram of a simple bus system is drawn in PWS, which is shown in Figure 2.3.

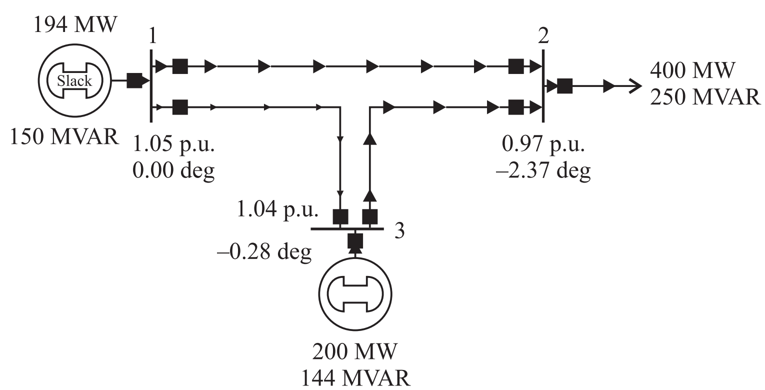
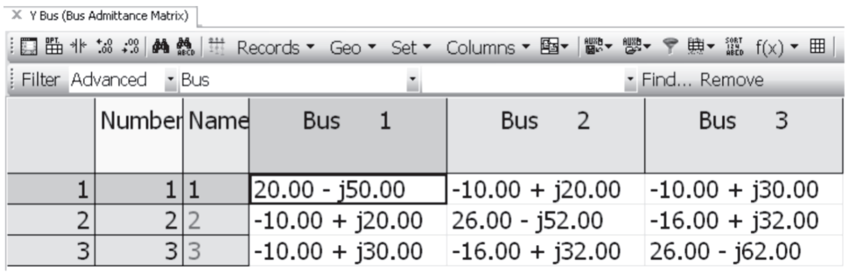


Figure 2.3 One line diagram of a simple three bus system (in PWS).

The first step is the formation of [Y_{bus}] using the inspection method. The calculated [Y_{bus}] values are given in Figure 2.4. Since the given problem is a three bus system, the size of [Y_{bus}] is 3 × 3 matrix.



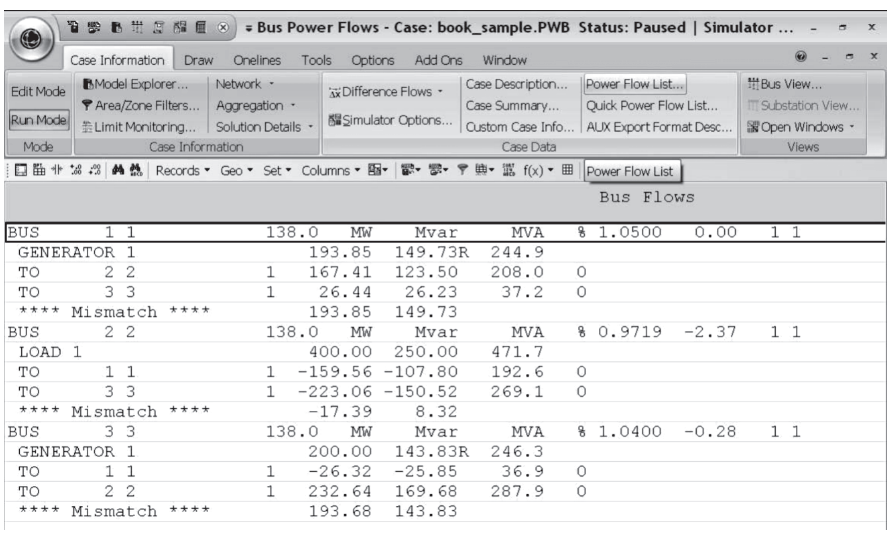
	Number	Name	Bus 1	Bus 2	Bus 3
1	1	1	20.00 - j50.00	-10.00 + j20.00	-10.00 + j30.00
2	2	2	-10.00 + j20.00	26.00 - j52.00	-16.00 + j32.00
3	3	3	-10.00 + j30.00	-16.00 + j32.00	26.00 - j62.00

Figure 2.4 Y_{bus} result.

There are three possible methods for executing load flow studies in Power World Simulator (PWS).

Gauss–Seidel method

Before executing this method, the number of iterations is to be fixed as 1 in *simulator options ribbon* to get the first iteration result. This method is executed by pressing the icon *Gauss–Seidel power flow* available in *tools ribbon*. The power flows and voltages are given in Figure 2.5 for the 1st iteration.



```
                                                            Bus Flows
BUS       1 1              138.0    MW      Mvar      MVA   % 1.0500    0.00    1 1
 GENERATOR 1                     193.85   149.73R  244.9
 TO       2 2              1     167.41   123.50   208.0   0
 TO       3 3              1      26.44    26.23    37.2   0
 **** Mismatch ****              193.85   149.73
BUS       2 2              138.0    MW      Mvar      MVA   % 0.9719   -2.37    1 1
 LOAD 1                          400.00   250.00   471.7
 TO       1 1              1   -159.56  -107.80   192.6   0
 TO       3 3              1   -223.06  -150.52   269.1   0
 **** Mismatch ****              -17.39     8.32
BUS       3 3              138.0    MW      Mvar      MVA   % 1.0400   -0.28    1 1
 GENERATOR 1                     200.00   143.83R  246.3
 TO       1 1              1    -26.32   -25.85    36.9   0
 TO       2 2              1    232.64   169.68   287.9   0
 **** Mismatch ****              193.68   143.83
```

Figure 2.5 Power flow results and voltages—1st iteration.

Now change the number of iterations as 2 in *simulator options ribbon* for getting the results of the second iteration and execute *Gauss–Seidel power flow*. The results are shown in Figure 2.6 for iteration 2.

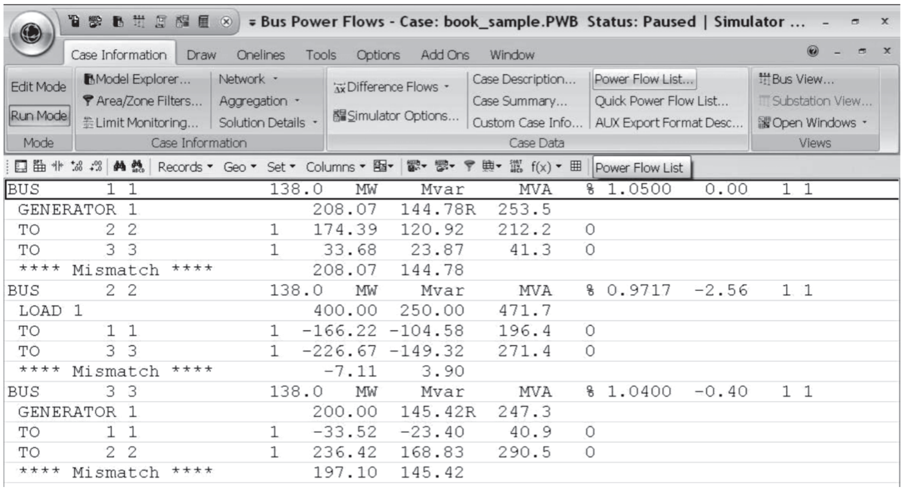


Figure 2.6 Power flow results and voltages—2nd iteration.

Figure 2.6 indicates that there are mismatches in all three bus voltages. So, the execution should be continued until converged solution is obtained. This method gives converged results after 8th iterations for this problem. Before executing the program, the numbers of iterations have to be changed as 10. This is shown in Figure 2.7.

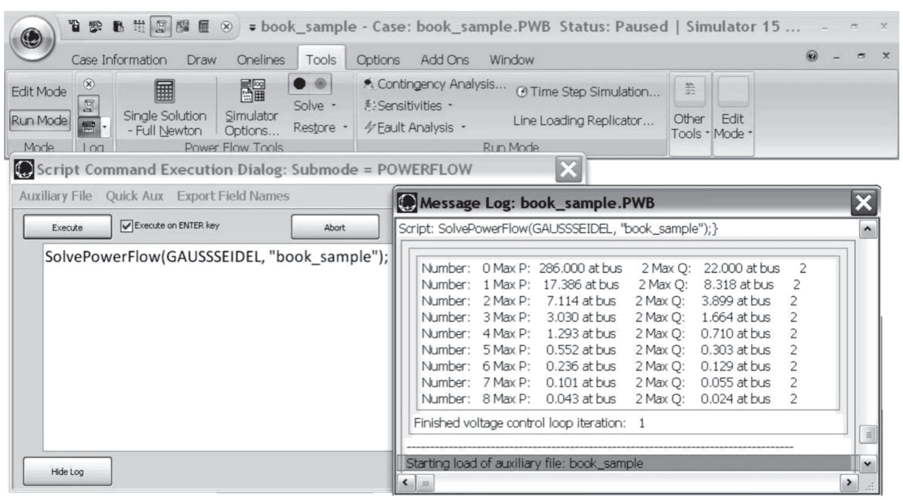


Figure 2.7 Details of convergence and iterations.

The final solutions are obtained after 8th iterations and it is shown in Figure 2.8.

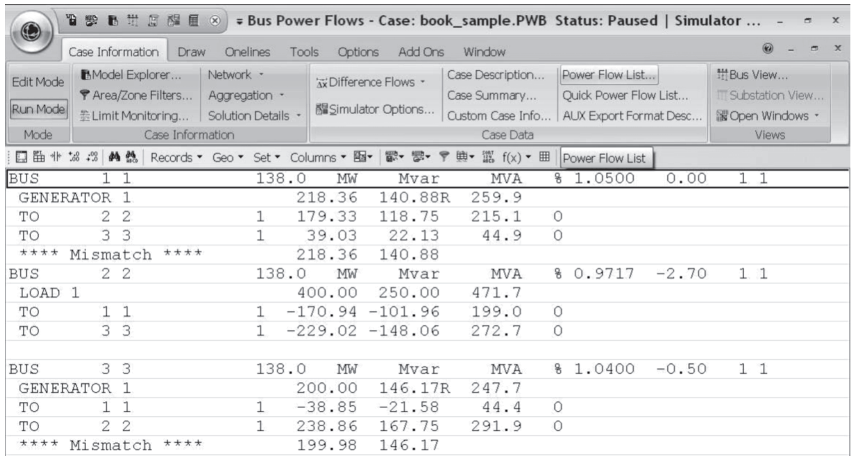


Figure 2.8 Converged power flow results and voltages.

PSS/E

The same problem is taken and drawn in PSS/E software and it is given in Figure 2.9.

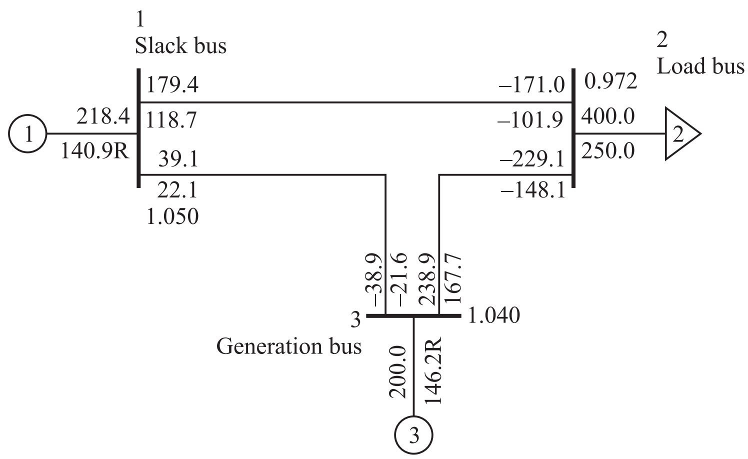


Figure 2.9 One line diagram of a simple three bus system (in PSS/E).

Once the data are entered in the software it can be executed by the above three power flow methods. Figure 2.10 shows the converged results obtained by the Gauss–Seidel method. This window is generated from *bus based report*.

2.5.2 Gauss–Seidel Method When *PV* Buses are Present

Some of the buses in an n bus power system are *PV* buses where P and V are specified but Q and δ are unknowns. The calculation strategy of Load flow solution with *PV* buses is different for *PQ* buses. Let the bus be numbered as

```
                PTI INTERACTIVE POWER            MON, JUN 27  2011    11:58    %MVA FOR TRANSFORMERS
              SYSTEM SIMULATOR--PSS®E                                 RATING   %I      FOR NON-TRANSFORMER BRANCHES
                                                                       SET A

BUS    1 SLACK BUS     CKT    MV     MVAR     MVA   %  1.0500PU   0.00   X---LOSSES---X  X---AREA----X  X---ZONE---X  1
  FROM GENERATION            218.4   140.9R   260.0  260      kV           MV    MVAR       1              1
  TO   2 LOAD BUS       1    179.3   118.8    215.1                        8.39  16.79      1              1
  TO   3 GEN BUS        1     39.1    22.2     44.9                        0.18   0.55      1              1

BUS    2 LOAD BUS      CKT    MV     MVAR     MVA   %  0.9717PU  –2.70   X---LOSSES---X  X---AREA----X  X---ZONE---X  2
                                                          kV               MV    MVAR       1              1
  TO LOAD-PQ                 400.0   250.0    471.7
  TO   1 SLACK BUS      1   –171.0  –102.0    199.1                        8.39  16.79      1              1
  TO   3 GEN BUS        1   –229.0  –148.1    272.7                        9.84  19.69      1              1

BUS    3 GEN BUS       CKT    MV     MVAR     MVA   %  1.0400PU  –0.50   X---LOSSES---X  X---AREA----X  X---ZONE---X  3
  FROM GENERATION            200.0   146.1R   247.7  248      kV           MV    MVAR       1              1
  TO   1 SLACK BUS      1    –38.9   –21.6     44.5                        0.18   0.55      1              1
  TO   2 LOAD BUS       1    238.9   167.8    291.8                        9.84  19.69      1              1
                                                                                                      Report
```

Figure 2.10 Converged results using Gauss–Seidel method.

$i = 1$ slack bus

$i = 2, 3, 4, ..., n$ PQ buses

$i = n + 1, n + 2, ..., n$ PV buses

Computational procedure

At the voltage controlled buses, bus voltages are specified and reactive power limits are also specified, i.e. $|V_i| = |V_i|_{spec}$; $Q_{i,\min} < Q_i < Q_{i,\max}$

1. Form the bus admittance matrix of the network by the direct inspection method, selecting the ground as reference [formation of Y_{bus}].
2. If slack bus is not specified, select one of the generator buses as the slack bus. The voltage at the slack bus is assumed as $V_i = V + j0.0$ [selection of the slack bus].
3. Assume initial values of voltages for all buses except the slack bus.

$$V_i^{(0)} = 1 + j0.0$$

4. For PV buses only angles $\delta_i^{(0)}$ have to be assumed.
5. Set convergence criterion = ε, i.e. if the largest of absolute of the residues exceeds the convergence criterion the process is repeated, otherwise it is terminated.
6. Set iteration count $k = 0$.
7. Bus count $i = 1$.
8. Check type of buses

   (a) If ith bus is PQ bus, go to step 10.
   (b) If ith bus is PV bus, go to the next step.

   Set $|V_i^k| = |V_i|_{spec}$
9. Calculate the reactive power of generator bus using the following equation

$$Q_i^{k+1} = -\,\text{im}\left[(V_i^k)^* \sum_{j=1}^{i-1} Y_{ij}(V_j)^{k+1} - (V_i^k)^* \sum_{j=1}^{n} Y_{ij}(V_j)^k\right] \tag{2.17}$$

   (a) If the calculated reactive power is within limits, then this bus can be treated as PV bus and set $Q_i = Q_i^{k+1}$.
   (b) If the calculated reactive power violates the limits, then this bus can be treated as PQ bus and set if

      (i) $Q_i^{k+1} < Q_{i,\min}$, then $Q_i = Q_{i,\min}$
      (ii) $Q_i^{k+1} > Q_{i,\max}$, then $Q_i = Q_{i,\max}$
10. Solve the voltage equation for bus i as, we know that

$$P_i - jQ_i = V_i^* \sum_{j=1}^{n} Y_{ij} V_j$$

$$\frac{P_i - jQ_i}{V_i^*} = V_i \sum_{j=0}^{n} Y_{ij} - \sum_{j=1}^{n} Y_{ij} V_j \qquad j \neq i$$

$$V_i = \frac{1}{Y_{ii}} \left[\frac{P_i - jQ_i}{V_i^*} - \sum_{\substack{j=1 \\ j \neq i}}^{n} Y_{ij} V_j \right]$$

or

$$V_i = \frac{1}{Y_{ii}} \left[\frac{P_i - jQ_i}{V_i^*} - \sum_{j=1}^{i-1} Y_{ij} V_j - \sum_{j=i+1}^{n} Y_{ij} V_j \right]$$

Equation (2.14) can be rewritten as

$$(V_i)^{k+1} = \frac{1}{Y_{ii}} \left[\frac{P_i - jQ_i}{(V_i^k)^*} - \sum_{j=1}^{i-1} Y_{ij} (V_j)^{k+1} - \sum_{j=i+1}^{n} Y_{ij} (V_j)^k \right]$$

11. Calculate the change in bus voltage

$$\Delta V_i^{k+1} = (V_i)^{k+1} - (V_i)^k$$

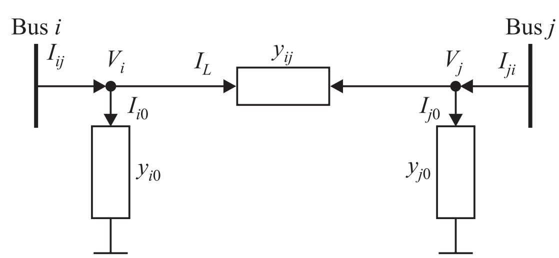
12. *Acceleration of convergence:* The process of convergence in the Gauss–Seidel method is slow as it requires larger number of iterations to obtain the solution. In this method, convergence can be increased by using the acceleration factor, denoted by α. In power flow studies, α is generally set about 1.6 and cannot exceed 2 if convergence is to occur. Therefore

$$V_{i,\text{acc}}^{k+1} = (V_i)^k + \alpha \, \Delta V_i^{k+1}$$

Calculate the bus voltages, i.e. V_i^{k+1} for all the buses except the slack bus, where i = 1, 2, 3, ..., n.

13. Repeat the iterating process until change in voltage (ΔV_i) for all the buses are within the specified or within the tolerance.
14. Finally calculate the power flow and power losses.

Current and power flows



$$i \rightarrow j$$

$$I_{ij} = I_L + I_{i0} = y_{ij}(V_i - V_j) + y_{i0} V_i$$

$$S_{ij} = V_i I_{ij}^* = V_i^2 (y_{ij} + y_{i0})^* - V_i y_{ij}^* V_i^*$$

$$j \rightarrow i$$

$$I_{ji} = I_L + I_{j0} = y_{ij}(V_j - V_i) + y_{j0}V_j$$

$$S_{ji} = V_j I_{ij}^* = V_j^2 (y_{ij} + y_{j0})^* - V_j y_{ij}^* V_i^*$$

Power loss

$$S_{\text{loss}ij} = S_{ij} + S_{ji}$$

This completes the load flow study. Finally, in Figure 2.11 all the computational steps are summarized in the detailed flow chart.

Advantages and Disadvantages of Gauss–Seidel Method

Advantages

- The calculations are simple and so there is less programming task to perform.
- The memory requirement is small.
- Useful for the small systems.

Disadvantages

- Requires a large number of iterations to converge.
- Not suitable for large systems.
- Convergence time increases with the size of the system.

EXAMPLE 2.4 A three-bus power system is shown in Figure 2.12. The system parameters are given in Table 2.4 and the generation and demand data in Table 2.5. The voltage at bus 2 is maintained at 1.04 p.u. The maximum and minimum reactive power limits of the generation at bus 2 are 35 and 0 MVAR respectively. Determine one iteration of the load flow solution using the Gauss–Seidel iterative method. Assume bus 1 as slack bus and acceleration factor $\alpha = 1.6$.

Table 2.4 Bus code and impedance

Bus code	*Impedance in* p.u.	*Bus code*	*Line charging admittance* $\frac{y'_{ij}}{2}$
1–2	0.06 + *j*0.18	1	*j*0.05
1–3	0.02 + *j*0.06	2	*j*0.06
2–3	0.04 + *j*0.12	3	*j*0.06

Table 2.5 Scheduled bus voltages, real and reactive powers of generation and demand

Bus no.	*Bus voltage*	*Generation*		*Demand*	
		MW	**MVAR**	**MW**	**MVAR**
1	1.06 + *j*0.0	—	—	0	0
2	1.04 + *j*0.0	20	—	0	0
3	—	0	0	60	25

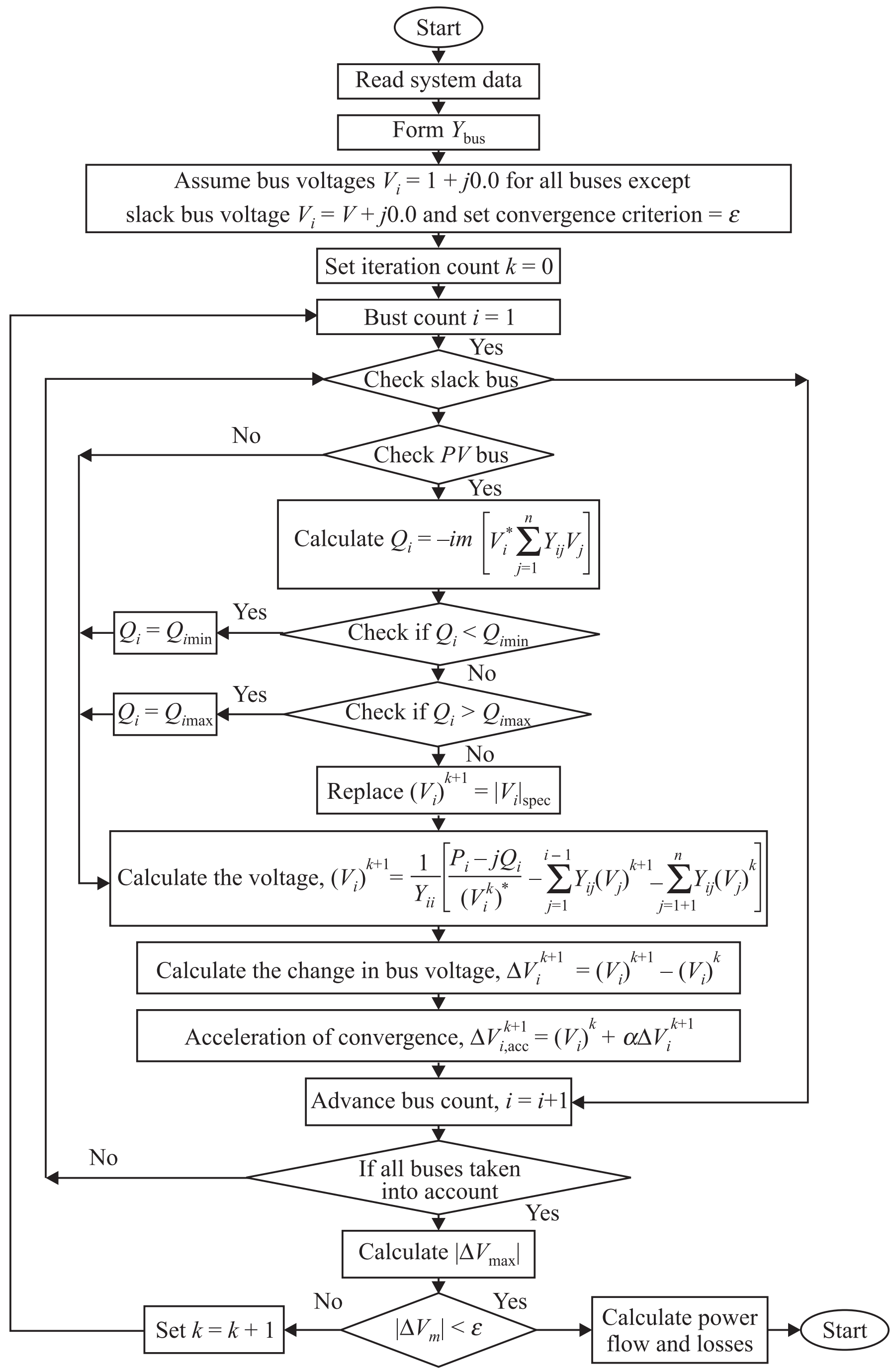


Figure 2.11 Flow chart for Gauss–Seidel method when PV buses are present.

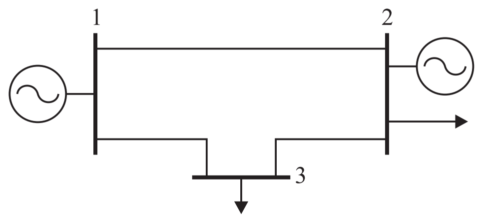


Figure 2.12 Three bus power system.

Solution: To form the Y_{bus}

$$y_{10} = \frac{y'_{12}}{2} + \frac{y'_{13}}{2} = j0.05 + j0.06 = j0.11$$

$$y_{20} = \frac{y'_{21}}{2} + \frac{y'_{22}}{2} = j0.05 + j0.06 = j0.11$$

$$y_{30} = \frac{y'_{31}}{2} + \frac{y'_{32}}{2} = j0.06 + j0.06 = j0.12$$

$$y_{12} = \frac{1}{z_{12}} = \frac{1}{0.06 + j0.18} = 1.67 - j5$$

$$y_{13} = \frac{1}{z_{13}} = \frac{1}{0.02 + j0.06} = 5 - j15$$

$$y_{23} = \frac{1}{z_{23}} = \frac{1}{0.04 + j0.12} = 2.5 - j7.5$$

$$Y_{11} = y_{10} + y_{12} + y_{13} = j0.11 + (1.67 - j5) + (5 - j15) = 6.67 - j19.89$$

$$Y_{12} = Y_{21} = -y_{12} = -(1.67 - j5) = -1.67 + j5$$

$$Y_{13} = Y_{31} = -y_{13} = -(5 - j15) = -5 + j15$$

$$Y_{22} = y_{20} + y_{21} + y_{23} = j0.11 + (1.67 - j5) + (2.5 - j7.5) = 4.17 - j12.39$$

$$Y_{23} = Y_{32} = -y_{23} = -(2.5 - j7.5) = -2.5 + j7.5$$

$$Y_{33} = y_{30} + y_{31} + y_{32} = j0.12 + (5 - j15) + (2.5 - j7.5) = 7.5 - j22.38$$

$$Y_{\text{bus}} = \begin{bmatrix} 6.67 - j19.89 & -1.67 + j5 & -5 + j15 \\ -1.67 + j5 & 4.17 - j12.39 & -2.5 + j7.5 \\ -5 + j15 & -2.5 + j7.5 & 7.5 - j22.38 \end{bmatrix}$$

At bus 2, $$P_2 = P_{G2} - P_{D2} = \frac{20}{100} - 0 = 0.2 \text{ p.u.}$$

$$Q_2 = Q_{G2} - Q_{D2} = ? - 0 = ? \text{ p.u.}$$

At bus 3, $$P_3 = P_{G3} - P_{D3} = 0 - \frac{60}{100} = -0.6 \text{ p.u.}$$

$$Q_3 = Q_{G3} - Q_{D3} = 0 - \frac{25}{100} = -0.25 \text{ p.u.}$$

First iteration

Set $k = 0$, Bus 1 is slack bus.

$$\therefore \quad V_1^0 = V_1^1 = V_1^2 = V_1^3 = \cdots = 1.06 + j0.0$$

$$V_2^0 = 1.04\angle 0$$

Assume a flat start voltage for PQ buses

$$V_3^0 = 1\angle 0$$

Determine the voltage at bus 2.

Given the reactive power limit:

$$0 \text{ MVAR} < Q_{G2} < 35 \text{ MVAR or } 0 \text{ p.u.} < Q_{G2} < 0.3 \text{ p.u.}$$

So to find V_2^1, first Q_2^1 is calculated.

$$(Q_i)^{k+1} = -\text{im}\left[(V_i^k)^* \sum_{j=1}^{i-1} Y_{ij}(V_j)^{k+1} - (V_i^k)^* \sum_{j=1}^{n} Y_{ij}(V_j)^k\right]$$

$$Q_2^1 = -\text{im}\,[(V_2^0)^* Y_{21} V_1^1 + (V_2^0)^* (Y_{22}V_2^0 + Y_{23}V_3^0)]$$

$$Q_2^1 = -\text{im}\,[(V_2^0)^* (Y_{21}V_1^1 + Y_{22}V_2^0 + Y_{23}V_3^0)]$$

$$Q_2^1 = -\text{im}\,[(1.04 - j0)\,(-1.67 + j5)\,(1.06 + j0.0) + (4.17 - j12.4)$$
$$(1.04 - j0) + (-2.5 + j7.5)\,(1 + j0.0)]$$

$$Q_2^1 = -\text{im}\,[0.06947 - j0.09984]$$

$$Q_2^1 = 0.09984$$

The value of Q_2^1 is within the limits and so the reactive power limit is not violated. Therefore bus 2 can be treated as PV bus.

Now to find V_2^1

$$V_2^1 = \frac{1}{Y_{22}}\left[\frac{P_2 - jQ_2^1}{(V_2^0)^*} - Y_{21}V_1^1 - Y_{23}V_3^0\right]$$

$$V_2^1 = \frac{1}{4.17 - j12.4}\left[\frac{0.2 - j0.09984}{1.04 - j0.0} - (-1.67 + j5)\,1.06 - (-2.5 + j7.5)\,(1.0)\right]$$

$$= 1.0432\angle 0.4985°$$

$$\delta_2^1 = \angle\, 0.4985°$$

We get $|V_2^1| = |V_2^1|_{\text{spec}}\angle\delta_2^1 = 1.01\angle 0.4985° = 1.0399 + j0.009$

The voltage at bus 3 (PQ bus) is

$$(V_3)^1 = \frac{1}{Y_{33}}\left[\frac{P_3 - jQ_3}{(V_3^0)^*} - Y_{31}V_2^1 - Y_{32}V_2^1\right]$$

$$V_3^1 = \frac{1}{7.5 - j22.38}\left[\frac{-0.6 + j0.25}{1 - j0} - (5 + j15)(1.06) - (-2.5 + j7.5)(1.0399 + j0.009)\right]$$

$$= 0.9499 + j0.0109$$

$$\Delta V_3^1 = (V_3)^1 - (V_3)^0 = (0.9499 + j0.0109) - (1 + j0.0)$$

$$= -0.0501 + j0.0109$$

$$V_{3,\text{acc}}^1 = (V_3)^0 + \alpha\,\Delta V_3^1 = (1 + j0.0) + 1.6 \times (-0.0501 + j0.0109)$$

$$= 0.91984 - j0.01744$$

The bus voltages at the end of the first iteration are

$$V_1^1 = 1.06 + j0$$

$$V_2^1 = 1.0399 + j0.009$$

$$V_3^1 = 0.91984 - j0.01744$$

EXAMPLE 2.5 If the reactive power constraint on generator 2 is 0.2 p.u. $< Q_{G2} <$ 0.5 p.u. in the Example 2.4, then find the bus voltages at the end of the first iteration. Assume the acceleration factor is 1.6.

Solution: In the previous example we have calculated Q_2^1 as

$$Q_2^1 = 0.09984$$

This value of reactive power violates the lower limit of Q_{G2}. Therefore Q_{G2} is fixed at 0.2 p.u. Hence the bus 2 is considered as a load bus.

Now

At bus 2, $$P_2 = P_{G2} - P_{D2} = \frac{20}{100} - 0 = 0.2 \text{ p.u.}$$

$$Q_2 = Q_{G2} - Q_{D2} = 0.2 - 0 = 0.2 \text{ p.u.}$$

At bus 3, $$P_3 = P_{G3} - P_{D3} = 0 - \frac{60}{100} = -0.6 \text{ p.u.}$$

$$Q_3 = Q_{G3} - Q_{D3} = 0 - \frac{25}{100} = -0.25 \text{ p.u.}$$

First iteration

Set $k = 0$, bus 1 is the slack bus.

$\therefore$ $$V_1^0 = V_1^1 = V_1^2 = V_1^3 = \cdots = 1.06 + j0.0$$

Assume a flat start voltage for PQ buses

$$V_2^0 = 1\angle 0$$

$$V_3^0 = 1\angle 0$$

The voltage at bus 2 (*PQ* bus) is

$$V_2^1 = \frac{1}{Y_{22}}\left[\frac{P_2 - jQ_2^1}{(V_2^0)^*} - Y_{21}V_1^1 - Y_{23}V_3^0\right]$$

$$V_2^1 = \frac{1}{4.17 - j12.4}\left[\frac{0.2 - j0.2}{(1 - j0.0)} - (-1.67 + j5)1.06 - (-2.5 + j7.5)(1.0)\right]$$

$$= 1.0508 + j0.00713$$

$$\Delta V_2^1 = (V_2)^1 - (V_2)^0 = (1.0508 + j0.00713) - (1 + j0.0)$$

$$= 0.0508 + j0.00713$$

$$V_{2,\text{acc}}^1 = (V_2)^0 + \alpha\,\Delta V_2^1 = (1 + j0.0) + 1.6 \times (0.0508 + j0.00713)$$

$$= 1.08128 + j0.0114$$

The voltage at bus 3 (*PQ* bus) is

$$(V_3)^1 = \frac{1}{Y_{33}}\left[\frac{P_3 - jQ_3}{(V_3^0)^*} - Y_{31}V_2^1 - Y_{32}V_2^1\right]$$

$$V_3^1 = \frac{1}{7.5 - j22.38}\left[\frac{-0.6 + j0.25}{1 - j0} - (5 + j15)(1.06) - (-2.5 + j7.5)(1.08128 + j0.0114)\right]$$

$$= 0.963 + j0.0117$$

$$\Delta V_3^1 = (V_3)^1 - (V_3)^0 = (0.963 + j0.0117) - (1 + j0.0)$$

$$= -0.037 + j0.0117$$

$$V_{3,\text{acc}}^1 = (V_3)^0 + \alpha\,\Delta V_3^1 = (1 + j0.0) + 1.6 \times (-0.037 + j0.0117)$$

$$= 0.9421 - j0.01872$$

The bus voltages at the end of the first iteration are

$$V_1^1 = 1.06 + j0$$

$$V_2^1 = 1.08128 + j0.0114$$

$$V_3^1 = 0.9421 - j0.01872$$

2.6 Newton–Raphson Load Flow Method

2.6.1 Introduction

The Newton–Raphson method is a competent algorithm to solve nonlinear equations. It transforms the procedure of solving nonlinear equations into the procedure of repeatedly solving linear equations. This sequential linearization process is the core of the Newton–Raphson method.

$$f(x) = 0 \tag{2.18}$$

Let us assume that $f(x)$ is continuous and differential at a point $x(0)$, the initial guess for the sought root. Assume the real solution x is close to $x(0)$,

$$x = x^{(0)} - \Delta x^{(0)} \tag{2.19}$$

where $\Delta x^{(0)}$ is a correction value of $x^{(0)}$. The following equation embraces to

$$f(x^{(0)} - \Delta x^{(0)}) = 0 \tag{2.20}$$

Now expanding the above equation in a Taylor series expansion about point $x^{(0)}$ yields:

$$f(x^{(0)} - \Delta x^{(0)}) = f(x^{(0)}) - f'(x^{(0)})\,\Delta x^{(0)} + f''(x^{(0)})\,\frac{(\Delta x^{(0)})^2}{2!} - \cdots + (-1)^n f^n(x^{(0)})\frac{(\Delta x^{(0)})^n}{n!} + \cdots = 0 \tag{2.21}$$

where $f'(x^{(0)}),\ \ldots, f^n(x^{(0)})$ are the different order partial derivatives of $f(x)$ at $x(0)$. If the initial guess is sufficiently close to the actual solution, the higher order terms of the Taylor series expansion could be neglected. Equation (2.21) becomes

$$f(x^{(0)}) - f'(x^{(0)})\,\Delta x^{(0)} = 0 \tag{2.22}$$

This is a linear equation in $\Delta x^{(0)}$ and can be easily solved.

Using $\Delta x^{(0)}$ to modify $x^{(0)}$, we can get $x^{(1)}$

$$x^{(1)} = x^{(0)} - \Delta x^{(0)} \tag{2.23}$$

$x^{(1)}$ may be close to the actual solution. Then using $x^{(1)}$ as the new guess value, we solve the following equation similar to Eq. (2.22)

$$f(x^{(1)}) - f'(x^{(1)})\,\Delta x^{(1)} = 0 \tag{2.24}$$

Thus $x^{(2)}$ is obtained.

$$x^{(2)} = x^{(1)} - \Delta x^{(1)} \tag{2.25}$$

Repeat this procedure to establish the correction equation in the kth iteration:

$$f(x^{(k)}) - f'(x^{(k)})\,\Delta x^{(k)} = 0 \tag{2.26}$$

or

$$f(x^{(k)}) = f'(x^{(k)})\,\Delta x^{(k)} \tag{2.27}$$

The left-hand of the above equation can be considered as the error produced by the approximate solution $x^{(k)}$. When $f(x^{(k)}) \Rightarrow 0$, Eq. (2.18) is satisfied, so $x^{(k)}$ is the solution of the equation.

Now we will extend the Newton's method to simultaneous nonlinear equations. Assume the nonlinear equations with variables $x_1, x_2, \ldots, x_n$:

$$\left.\begin{aligned} f_1(x_1, x_2, \ldots, x_n) &= 0 \\ f_2(x_1, x_2, \ldots, x_n) &= 0 \\ &\vdots \\ f_n(x_1, x_2, \ldots, x_n) &= 0 \end{aligned}\right\} \tag{2.28}$$

Specify the initial guess values of all variables $x_1^{(0)}, x_2^{(0)}, \ldots, x_n^{(0)}$. Let $\Delta x_1^{(0)}$, $\Delta x_2^{(0)}$, $\Delta x_3^{(0)}$, ..., $\Delta x_n^{(0)}$ be the correction values to satisfy the following equations:

$$\left.\begin{aligned} f_1(x_1^{(0)} - \Delta x_1^{(0)}, x_2^{(0)} - \Delta x_2^{(0)}, \ldots, x_n^{(0)} - \Delta x_n^{(0)}) = 0 \\ f_2(x_1^{(0)} - \Delta x_1^{(0)}, x_2^{(0)} - \Delta x_2^{(0)}, \ldots, x_n^{(0)} - \Delta x_n^{(0)}) = 0 \\ \vdots \\ f_n(x_1^{(0)} - \Delta x_1^{(0)}, x_2^{(0)} - \Delta x_2^{(0)}, \ldots, x_n^{(0)} - \Delta x_n^{(0)}) = 0 \end{aligned}\right\} \tag{2.29}$$

Expanding the above equations via the multivariate Taylor series and neglecting the higher order terms, we have the following equations:

$$\left.\begin{aligned} f_1(x_1^{(0)}, x_2^{(0)}, \ldots, x_n^{(0)})\left[\left(\frac{\partial f_1}{\partial x_1}\right)^{(0)} \Delta x_1^{(0)} + \left(\frac{\partial f_1}{\partial x_2}\right)^{(0)} \Delta x_2^{(0)} + \cdots + \left(\frac{\partial f_1}{\partial x_n}\right)^{(0)} \Delta x_n^{(0)}\right] = 0 \\ f_2(x_1^{(0)}, x_2^{(0)}, \ldots, x_n^{(0)})\left[\left(\frac{\partial f_2}{\partial x_1}\right)^{(0)} \Delta x_1^{(0)} + \left(\frac{\partial f_2}{\partial x_2}\right)^{(0)} \Delta x_2^{(0)} + \cdots + \left(\frac{\partial f_2}{\partial x_n}\right)^{(0)} \Delta x_n^{(0)}\right] = 0 \\ \vdots \qquad\qquad\qquad \vdots \qquad\qquad\qquad \\ f_n(x_1^{(0)}, x_2^{(0)}, \ldots, x_n^{(0)})\left[\left(\frac{\partial f_n}{\partial x_1}\right)^{(0)} \Delta x_1^{(0)} + \left(\frac{\partial f_n}{\partial x_2}\right)^{(0)} \Delta x_2^{(0)} + \cdots + \left(\frac{\partial f_n}{\partial x_n}\right)^{(0)} \Delta x_n^{(0)}\right] = 0 \end{aligned}\right\} \tag{2.30}$$

Here $(\partial f_1/\partial x_2)^{(0)}$ is the partial derivative of function $f_1(x_1, x_2, \ldots, x_n)$ over independent variable x_j at the point $(x_1^{(0)}, x_2^{(0)}, \ldots, x_n^{(0)})$. Rewrite the above equation in the matrix form.

$$\begin{bmatrix} f_1(x_1^{(0)}, x_2^{(0)}, \ldots, x_n^{(0)}) \\ f_2(x_1^{(0)}, x_2^{(0)}, \ldots, x_n^{(0)}) \\ \vdots \qquad \vdots \\ f_n(x_1^{(0)}, x_2^{(0)}, \ldots, x_n^{(0)}) \end{bmatrix} = \begin{bmatrix} \left(\frac{\partial f_1}{\partial x_1}\right)^{(0)} & \left(\frac{\partial f_1}{\partial x_2}\right)^{(0)} & \cdots & \left(\frac{\partial f_1}{\partial x_n}\right)^{(0)} \\ \left(\frac{\partial f_2}{\partial x_1}\right)^{(0)} & \left(\frac{\partial f_2}{\partial x_2}\right)^{(0)} & \cdots & \left(\frac{\partial f_2}{\partial x_n}\right)^{(0)} \\ \vdots & \vdots & \cdots & \vdots \\ \left(\frac{\partial f_n}{\partial x_1}\right)^{(0)} & \left(\frac{\partial f_n}{\partial x_2}\right)^{(0)} & \cdots & \left(\frac{\partial f_n}{\partial x_n}\right)^{(0)} \end{bmatrix} \begin{bmatrix} \Delta x_1^{(0)} \\ \Delta x_2^{(0)} \\ \vdots \\ \Delta x_n^{(0)} \end{bmatrix} \tag{2.31}$$

After solving $\Delta x_1^{(0)}, \Delta x_2^{(0)}, \cdots, \Delta x_n^{(0)}$ from the above equation, we get

$$\left.\begin{aligned} x_1^{(1)} &= x_1^{(0)} - \Delta x_1^{(0)} \\ x_2^{(1)} &= x_2^{(0)} - \Delta x_2^{(0)} \\ &\vdots \\ x_n^{(1)} &= x_n^{(0)} - \Delta x_n^{(0)} \end{aligned}\right\} \tag{2.32}$$

$x_1^{(1)}, x_2^{(1)}, \ldots, x_n^{(1)}$ will approach the actual solution more closely. The updated values are used as the new guess to solve the correction equation (2.31) and to further correct the variables. In this way the iterative process of the Newton–Raphson method is formed.

Generally, the correction in the kth iteration can be written as

$$\begin{bmatrix} f_1(x_1^{(k)}, x_2^{(k)}, \ldots, x_n^{(k)}) \\ f_2(x_1^{(k)}, x_2^{(k)}, \ldots, x_n^{(k)}) \\ \vdots \\ f_n(x_1^{(k)}, x_2^{(k)}, \ldots, x_n^{(k)}) \end{bmatrix} = \begin{bmatrix} \left(\frac{\partial f_1}{\partial x_1}\right)^{(k)} & \left(\frac{\partial f_1}{\partial x_2}\right)^{(k)} & \cdots & \left(\frac{\partial f_1}{\partial x_n}\right)^{(k)} \\ \left(\frac{\partial f_2}{\partial x_1}\right)^{(k)} & \left(\frac{\partial f_2}{\partial x_2}\right)^{(k)} & \cdots & \left(\frac{\partial f_2}{\partial x_n}\right)^{(k)} \\ \vdots & \vdots & \cdots & \vdots \\ \left(\frac{\partial f_n}{\partial x_1}\right)^{(k)} & \left(\frac{\partial f_n}{\partial x_2}\right)^{(k)} & \cdots & \left(\frac{\partial f_n}{\partial x_n}\right)^{(k)} \end{bmatrix} \begin{bmatrix} \Delta x_1^{(k)} \\ \Delta x_2^{(k)} \\ \vdots \\ \Delta x_n^{(k)} \end{bmatrix} \tag{2.33}$$

The above equation can be expressed in the matrix form as

$$\mathbf{F} = \mathbf{JC} \tag{2.34}$$

where,

$$\mathbf{F} = \begin{bmatrix} f_1(x_1^{(k)}, x_2^{(k)}, \ldots, x_n^{(k)}) \\ f_2(x_1^{(k)}, x_2^{(k)}, \ldots, x_n^{(k)}) \\ \vdots \\ f_n(x_1^{(k)}, x_2^{(k)}, \ldots, x_n^{(k)}) \end{bmatrix} \tag{2.35}$$

is the error vector in the kth iteration.

$$\mathbf{J} = \begin{bmatrix} \left(\frac{\partial f_1}{\partial x_1}\right)^{(k)} & \left(\frac{\partial f_1}{\partial x_2}\right)^{(k)} & \cdots & \left(\frac{\partial f_1}{\partial x_n}\right)^{(k)} \\ \left(\frac{\partial f_2}{\partial x_1}\right)^{(k)} & \left(\frac{\partial f_2}{\partial x_2}\right)^{(k)} & \cdots & \left(\frac{\partial f_2}{\partial x_n}\right)^{(k)} \\ \vdots & \vdots & \cdots & \vdots \\ \left(\frac{\partial f_n}{\partial x_1}\right)^{(k)} & \left(\frac{\partial f_n}{\partial x_2}\right)^{(k)} & \cdots & \left(\frac{\partial f_n}{\partial x_n}\right)^{(k)} \end{bmatrix} \tag{2.36}$$

is the first derivative matrix and it is called **Jacobian matrix**.

$$\mathbf{C} = \begin{bmatrix} \Delta x_1^{(k)} \\ \Delta x_2^{(k)} \\ \vdots \\ \Delta x_n^{(k)} \end{bmatrix} \tag{2.37}$$

is the correction value vector in the kth iteration.

We also have the equation similar to Eq. (2.32)

$$X^{(k+1)} = X^{(k)} - \Delta X^{(k)} \tag{2.38}$$

The state update vector $\Delta X^{(k)}$ is calculated from Eq. (2.33) by taking the inverse of the Jacobian matrix. Thus we get

$$X^{(k)} = -[J]^{-1}F \tag{2.39}$$

With Eqs. (2.34) and (2.38) solved alternately in each iteration, $X^{(k+1)}$ gradually approaches the actual solution. Convergence can be evaluated by the norm of the correction value,

$$|\Delta X^{(k)}| < \varepsilon \tag{2.40}$$

2.6.2 Load Flow Solution Using Newton–Raphson Method

For large interconnected power systems among the numerous solution methods available for load flow analysis, the Newton–Raphson method is considered to be the most important. Many advantages are attributed to the Newton–Raphson approach. Its convergence characteristics are relatively powerful compared to the alternative processes, and very low computing times are achieved when sparse network equations are solved by the technique of sparsity programmed ordered elimination. The reliability of the Newton–Raphson method is comparatively good, since it can solve cases that lead to divergence with the other popular processes, but the method is by no means reliable. Failure does not occur on some ill-conditioned problems.

The number of iterations required to obtain a solution is independent of the system size, but more functional evaluations are required at each iteration. Since in the load flow problem real power and magnitude of bus voltage are specified for the PV buses, the load flow equation is formulated in the polar form.

The load flow equations can be rewritten as follows.

Real power

$$P_i^{(k)} = |V_i| \sum_{j=1}^{n} |V_j||Y_{ij}| \cos(\theta_{ij} + \delta_j - \delta_i) \tag{2.41}$$

Reactive power

$$Q_i^{(k)} = -|V_i| \sum_{j=1}^{n} |V_j||Y_{ij}| \sin(\theta_{ij} + \delta_j - \delta_i) \tag{2.42}$$

We have two equations for each load bus, given by Eqs. (2.41) and (2.42), and one equation for each voltage controlled bus, given by Eq. (2.41). Expanding Eqs. (2.41) and (2.42) in Taylor's series about the initial estimate and neglecting all higher order terms result in the following set of linear equations.

$$\begin{bmatrix} \Delta P_2^{(k)} \\ \vdots \\ \Delta P_n^{(k)} \\ \Delta Q_2^{(k)} \\ \vdots \\ \Delta Q_n^{(k)} \end{bmatrix} = \begin{bmatrix} \frac{\partial P_2^{(k)}}{\partial \delta_2} & \cdots & \frac{\partial P_2^{(k)}}{\partial \delta_n} & \frac{\partial P_2^{(k)}}{\partial |V_2|} & \cdots & \frac{\partial P_2^{(k)}}{\partial |V_n|} \\ \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\ \frac{\partial P_n^{(k)}}{\partial \delta_2} & \cdots & \frac{\partial P_n^{(k)}}{\partial \delta_n} & \frac{\partial P_n^{(k)}}{\partial |V_2|} & \cdots & \frac{\partial P_n^{(k)}}{\partial |V_n|} \\ \frac{\partial Q_2^{(k)}}{\partial \delta_2} & \cdots & \frac{\partial Q_2^{(k)}}{\partial \delta_n} & \frac{\partial Q_2^{(k)}}{\partial |V_2|} & \cdots & \frac{\partial Q_2^{(k)}}{\partial |V_n|} \\ \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\ \frac{\partial Q_n^{(k)}}{\partial \delta_2} & \cdots & \frac{\partial Q_n^{(k)}}{\partial \delta_n} & \frac{\partial Q_n^{(k)}}{\partial |V_2|} & \cdots & \frac{\partial Q_n^{(k)}}{\partial |V_n|} \end{bmatrix} \begin{bmatrix} \Delta \delta_2^{(k)} \\ \vdots \\ \Delta \delta_n^{(k)} \\ \Delta |V_2^{(k)}| \\ \vdots \\ \Delta |V_n^{(k)}| \end{bmatrix} \quad (2.43)$$

In the above equation, bus 1 is assumed to be the slack bus. The Jacobian matrix gives the linearised relationship between small changes in voltage angle $\Delta\delta_i^{(k)}$ and voltage magnitude $\Delta|V_i^{(k)}|$ with the small changes in real and reactive powers $\Delta P_i^{(k)}$ and $\Delta Q_i^{(k)}$ respectively. The elements of the Jacobian matrix are the partial derivatives of Eqs. (2.41) and (2.42), calculated at $\Delta\delta_i^{(k)}$ and $\Delta|V_2^{(k)}|$.

The above equation can be written as

$$\begin{bmatrix} \Delta P \\ \Delta Q \end{bmatrix} = \begin{bmatrix} J_1 & J_2 \\ J_3 & J_4 \end{bmatrix} \begin{bmatrix} \Delta\delta \\ \Delta|V| \end{bmatrix} \quad (2.44)$$

For the *PV* buses, the voltage magnitudes are known. Therefore, if *m* buses of the system are voltage controlled equations involving ΔQ and ΔV, and the corresponding columns of the Jacobian matrix are eliminated, then there are $(n - 1)$ real power constraints and $(n - 1 - m)$ reactive power constraints, and the order of the complete Jacobian matrix is $(2n - 2 - m) \times (2n - 2 - m)$.

Order of Jacobian matrix $\mathbf{J}_1$ is $(n - 1) \times (n - 1)$.
Order of Jacobian matrix $\mathbf{J}_2$ is $(n - 1) \times (n - 1 - m)$.
Order of Jacobian matrix $\mathbf{J}_3$ is $(n - 1 - m) \times (n - 1)$.
Order of Jacobian matrix $\mathbf{J}_4$ is $(n - 1 - m) \times (n - 1 - m)$.

Elements of Jacobian matrix $\mathbf{J}_1$

(i) the diagonal elements are

$$\frac{\partial P_i}{\partial \delta_i} = \sum_{j \neq i} |V_i||V_j||Y_{ij}| \sin(\theta_{ij} - \delta_i + \delta_j) \quad (2.45)$$

(ii) the off-diagonal elements are

$$\frac{\partial P_i}{\partial \delta_j} = -|V_i||V_j||Y_{ij}| \sin(\theta_{ij} - \delta_i + \delta_j) \qquad j \neq i \quad (2.46)$$

Elements of Jacobian matrix $\mathbf{J}_2$

(i) the diagonal elements are

$$\frac{\partial P_i}{\partial |V_i|} = 2|V_i||Y_{ii}|\cos\theta_{ii} + \sum_{j \neq i} |V_j||Y_{ij}|\cos(\theta_{ij} - \delta_i + \delta_j) \tag{2.47}$$

(ii) the off-diagonal elements are

$$\frac{\partial P_i}{\partial |V_j|} = |V_i||Y_{ij}|\cos(\theta_{ij} - \delta_i + \delta_j) \qquad j \neq i \tag{2.48}$$

Elements of Jacobian matrix $\mathbf{J}_3$

(i) the diagonal elements are

$$\frac{\partial Q_i}{\partial \delta_i} = \sum_{j \neq i} |V_i||V_j||Y_{ij}|\cos(\theta_{ij} - \delta_i + \delta_j) \tag{2.49}$$

(ii) the off-diagonal elements are

$$\frac{\partial Q_i}{\partial \delta_j} = -|V_i||V_j||Y_{ij}|\cos(\theta_{ij} - \delta_i + \delta_j) \qquad j \neq i \tag{2.50}$$

Elements of Jacobian matrix $\mathbf{J}_4$

(i) the diagonal elements are

$$\frac{\partial Q_i}{\partial |V_i|} = -2|V_i||Y_{ii}|\sin\theta_{ii} - \sum_{j \neq i} |V_j||Y_{ij}|\sin(\theta_{ij} - \delta_i + \delta_j) \tag{2.51}$$

(ii) the off-diagonal elements are

$$\frac{\partial Q_i}{\partial |V_j|} = -|V_i||Y_{ij}|\sin(\theta_{ij} - \delta_i + \delta_j) \qquad j \neq i \tag{2.52}$$

Difference in scheduled to calculated power (power residuals) is given by

$$\Delta P_i^{[k]} = P_{i,\text{sch}} - P_i^{[k]} \tag{2.53}$$

$$\Delta Q_i^{[k]} = Q_{i,\text{sch}} - Q_i^{[k]} \tag{2.54}$$

The new estimates for the voltage magnitude and angle

$$\delta_i^{[k+1]} = \delta_i^{[k]} + \Delta\delta_i^{[k]} \tag{2.55}$$

$$|V_i^{[k+1]}| = |V_i^{[k]}| + \Delta|V_i^{[k]}| \tag{2.56}$$

Computation procedure

1. Set flat start

   - For load buses, set the voltages equal to the slack bus or $1\angle 0°$.
   - For generator buses, set the angles equal to the slack bus or $0°$.

2. Calculate power mismatch
   - For load buses, calculate $P_i^{[k]}$ (Eq. (2.41)) and $Q_i^{[k]}$ (Eq. (2.42)) injections using the known and estimated system voltages.
   - For generator buses, calculate $P_i^{[k]}$ (Eq. (2.41)) and $\Delta P_i^{[k]}$ (Eq. (2.53)).
3. Form the Jacobian matrix
   - Use the various equations for the partial derivatives with respect to the voltage angle and magnitudes (form the Jacobian matrix).
   - The elements of Jacobian matrix ($\mathbf{J}_1$, $\mathbf{J}_2$, $\mathbf{J}_3$ and $\mathbf{J}_4$) calculated from Eqs. (2.45) to (2.52).
4. Find the matrix solution
   - Inverse the Jacobian matrix and multiply by the mismatch power.
   - Compute $\Delta\delta$ and $\Delta|V|$.
5. Difference in scheduled to calculated power

$$\Delta P_i^{[k]} = P_{i,\mathrm{sch}} - P_i^{[k]}$$
$$\Delta Q_i^{[k]} = Q_{i,\mathrm{sch}} - Q_i^{[k]}$$

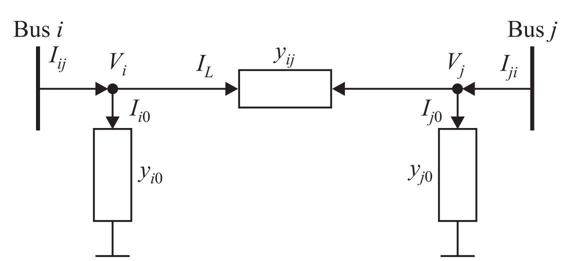
6. Find the new estimates for the voltage magnitude and angle

$$\delta_i^{[k+1]} = \delta_i^{[k]} + \Delta\delta_i^{[k]}$$
$$|V_i^{[k+1]}| = |V_i^{[k]}| + \Delta|V_i^{[k]}|$$

7. Repeat the process until the mismatch (residuals) is less than the specified accuracy

$$|\Delta P_i^{[k]}| \le \varepsilon$$
$$|\Delta Q_i^{[k]}| \le \varepsilon$$

8. After solving for bus voltages and angles, power flows and losses on the network branches are calculated
   - Transmission lines and transformers are network branches.
   - The direction of positive current flow is defined for a branch element (demonstrated on a medium length line).
   - Power flow is defined for each end of the branch.
   - Example: The power leaving bus *i* and flowing to bus *j* as shown below.



Current and power flows

$$i \rightarrow j$$

$$I_{ij} = I_L + I_{i0} = y_{ij}(V_i - V_j) + y_{i0}V_i$$

$$S_{ij} = V_i I_{ij}^* = V_i^2 (y_{ij} + y_{i0})^* - V_i y_{ij}^* V_i^*$$

$$j \rightarrow i$$

$$I_{ji} = -I_L + I_{j0} = y_{ij}(V_j - V_i) + y_{j0}V_j$$

$$S_{ji} = V_j I_{ij}^* = V_j^2 (y_{ij} + y_{j0})^* - V_j y_{ij}^* V_i^*$$

Power loss

$$S_{\text{loss}ij} = S_{ij} + S_{ji}$$

This completes the load flow study. Finally, in Figure 2.13 all the computational steps are summarized in the detailed flow chart.

2.6.3 Advantages and Disadvantages of Newton–Raphson Method

Advantages

Faster, more reliable and yields accurate results, requires less number of iterations.

Disadvantages

Program as well as memory is more complex.

EXAMPLE 2.6 Figure 2.14 shows the one line diagram of a simple three-bus system with generation at bus 1. The magnitude of voltage at bus 1 is adjusted to 1.05 p.u. The scheduled loads at buses 2 and 3 are given in the diagram. Line impedances are marked in p.u. on a 100 MVA base and the line charging susceptances are neglected.

(a) Using the Newton–Raphson method, determine the phasor values of the voltages at the load buses 2 and 3(*PQ* buses) accurate to decimal places.
(b) Verify the result with Power World Simulator.

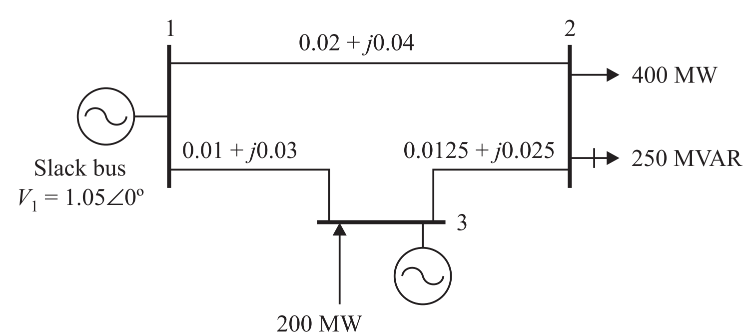


Figure 2.14 One line diagram of a simple three-bus system Example 2.6.

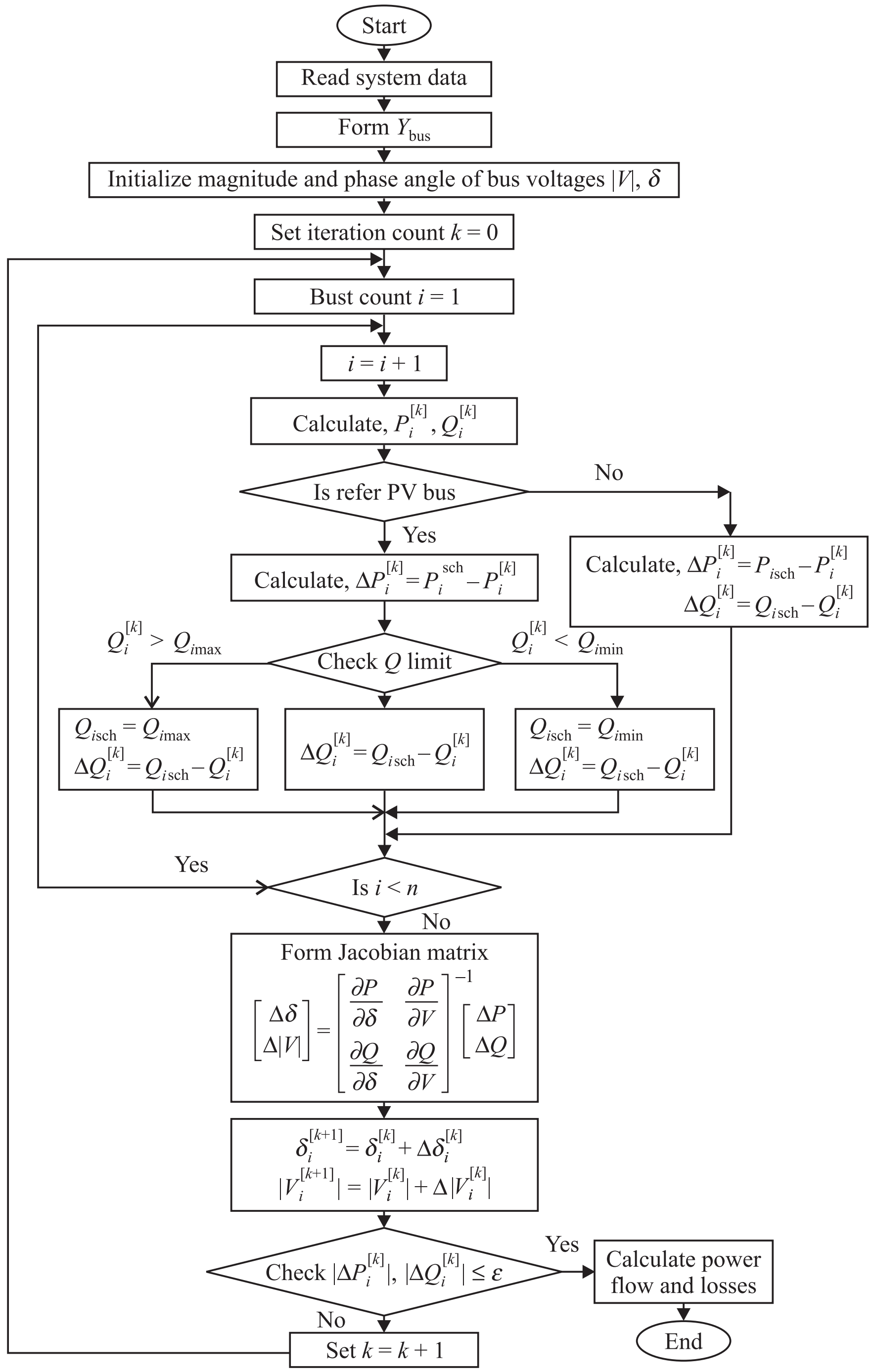


Figure 2.13 Flow chart for Newton–Raphson method.

Solution:

(a) Form the Y_{bus}

$$y_{12} = \frac{1}{z_{12}} = \frac{1}{0.02 + j0.04} = 10 - j20$$

$$y_{13} = \frac{1}{z_{13}} = \frac{1}{0.01 + j0.03} = 10 - j30$$

$$y_{23} = \frac{1}{z_{23}} = \frac{1}{0.0125 + j0.025} = 16 - j32$$

$$Y_{11} = y_{12} + y_{13} = (10 - j20) + (10 - j30) = 20 - j50$$

$$Y_{12} = Y_{21} = -y_{12} = -(10 - j20) = -10 + j20$$

$$Y_{13} = Y_{31} = -y_{13} = -(10 - j30) = -10 + j30$$

$$Y_{22} = y_{21} + y_{23} = (10 - j20) + (16 - j32) = 26 - j52$$

$$Y_{23} = Y_{32} = -y_{23} = -(16 - j32) = -16 + j32$$

$$Y_{33} = y_{31} + y_{32} = (10 - j30) + (16 - j32) = 26 - j62$$

$$Y_{bus} = \begin{bmatrix} 20 - j50 & -10 + j20 & -10 + j30 \\ -10 + j20 & 26 - j52 & -16 + j32 \\ -10 + j30 & -16 + j32 & 26 - j62 \end{bmatrix}$$

$$Y_{bus} = \begin{bmatrix} 53.85165\angle -1.9029 & 22.36068\angle 2.0344 & 31.62278\angle 1.8925 \\ 22.36068\angle 2.0344 & 58.13777\angle -1.1071 & 35.77709\angle 2.0344 \\ 31.62278\angle 1.8925 & 35.77709\angle 2.0344 & 67.23095\angle -1.1737 \end{bmatrix}$$

$$Y_{bus} = \begin{bmatrix} 53.85165\angle -68.2 & 22.36068\angle 116.6 & 31.62278\angle 108.4 \\ 22.36068\angle 116.6 & 58.13777\angle -63.4 & 35.77709\angle 116.6 \\ 31.62278\angle 108.4 & 35.77709\angle 116.6 & 67.23095\angle -67.2 \end{bmatrix}$$

Initialize magnitude and angle of bus voltage

$$|V_1| = 1.05,\ \delta_1 = 0.0 \text{ rad}$$

$$|V_2|^{(0)} = 1,\ \delta_2^{(0)} = 0.0 \text{ rad}$$

$$|V_3|^{(0)} = 1.04,\ \delta_3^{(0)} = 0.0 \text{ rad}$$

In the matrix form

$$\begin{bmatrix} \delta_1^{(0)} \\ V_1^{(0)} \end{bmatrix} = \begin{bmatrix} 0 \\ 1.05 \end{bmatrix};\ \begin{bmatrix} \delta_2^{(0)} \\ V_2^{(0)} \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \end{bmatrix};\ \begin{bmatrix} \delta_3^{(0)} \\ V_3^{(0)} \end{bmatrix} = \begin{bmatrix} 0 \\ 1.04 \end{bmatrix}$$

Scheduled powers are

At bus 2, $P_{2,sch} = P_{G2} - P_{D2} = 0 - \frac{400}{100} = -4 \text{ p.u.}$

$$Q_{2,sch} = Q_{G2} - Q_{D2} = 0 - \frac{250}{100} = -2.5 \text{ p.u.}$$

At bus 3, $P_{3,sch} = P_{G3} - P_{D3} = \frac{200}{100} - 0 = 2 \text{ p.u.}$

The real power at buses 2 and 3 and reactive power at bus 2 are

$$P_2 = |V_2||V_1||Y_{21}| \cos(\theta_{21} - \delta_2 + \delta_1) + |V_2^2||Y_{22}| \cos\theta_{22} + |V_2||V_3||Y_{23}| \cos(\theta_{23} - \delta_2 + \delta_3)$$

$$P_2 = (1)\ (1.05)\ (22.36068) \cos(116.6 - 0 + 0) + (1)^2 (58.13777) \cos(-63.4) + (1)\ (1.04)\ (35.77709) \cos(116.6 - 0 + 0) = -1.1414$$

$$P_3 = |V_3||V_1||Y_{31}| \cos(\theta_{31} - \delta_3 + \delta_1) + |V_3||V_2||Y_{32}| \cos(\theta_{32} - \delta_3 + \delta_2) + |V_3^2||Y_{33}| \cos\theta_{33}$$

$$P_3 = (1.04)\ (1.05)\ (31.62278) \cos(108.4 - 0 + 0) + (1.04)(1)(35.77709) \cos(116.6 - 0 + 0) + (1.04)^2 (67.23095) \cos(-67.2) = 0.5616$$

$$Q_2 = -|V_2||V_1||Y_{21}| \sin(\theta_{21} - \delta_2 + \delta_1) - |V_2^2||Y_{22}| \sin\theta_{22} - |V_2||V_3||Y_{23}| \sin(\theta_{23} - \delta_2 + \delta_3)$$

$$Q_2 = -(1)\ (1.05)\ (22.36068) \sin(116.6 - 0 + 0) - (1)^2 (58.13777) \sin(-63.4) - (1)\ (1.04)\ (35.77709) \sin(116.6 - 0 + 0) = -2.28$$

Difference in scheduled to calculated power

$$\Delta P_2^{[0]} = P_{2,\text{sch}} - P_{2,\text{calc}}^{(0)} = -4 - (-1.1414) = -2.8586$$

$$\Delta P_3^{[0]} = P_{3,\text{sch}} - P_{3,\text{calc}}^{(0)} = 2 - (0.5616) = 1.43846$$

$$\Delta Q_2^{[0]} = Q_{2,\text{sch}} - Q_{2,\text{calc}}^{(0)} = -2.5 - (-2.28) = -0.22$$

The Jacobian matrix is given by

$$\begin{bmatrix} \Delta P_2 \\ \Delta P_3 \\ \Delta Q_2 \end{bmatrix} = \begin{bmatrix} \dfrac{\partial P_2}{\partial \delta_2} & \dfrac{\partial P_2}{\partial \delta_3} & \dfrac{\partial P_2}{\partial |V_2|} \\ \dfrac{\partial P_3}{\partial \delta_2} & \dfrac{\partial P_3}{\partial \delta_3} & \dfrac{\partial P_3}{\partial |V_2|} \\ \dfrac{\partial Q_2}{\partial \delta_2} & \dfrac{\partial Q_2}{\partial \delta_3} & \dfrac{\partial Q_2}{\partial |V_2|} \end{bmatrix} \begin{bmatrix} \Delta \delta_2 \\ \Delta \delta_3 \\ \Delta |V_2| \end{bmatrix}$$

$$\frac{\partial P_2}{\partial \delta_2} = |V_2||V_1||Y_{21}| \sin(\theta_{21} - \delta_2 + \delta_1) + |V_2||V_3||Y_{23}| \sin(\theta_{23} - \delta_2 + \delta_3)$$

$$= (1)\ (1.05)\ (22.36068) \sin(116.6 - 0 + 0) + (1)\ (1.04)\ (35.77709) \sin(116.6 - 0 + 0) = 54.2634$$

$$\frac{\partial P_2}{\partial \delta_3} = -|V_1||V_3||Y_{23}| \sin(\theta_{23} - \delta_2 + \delta_3)$$

$$= -(1)\ (1.04)\ (35.77709) \sin(116.6 - 0 + 0) = -33.2698$$

$$\frac{\partial P_2}{\partial |V_2|} = |V_1||Y_{21}| \cos(\theta_{21} - \delta_2 + \delta_1) + 2|V_2||Y_{22}| \cos\theta_{22} + |V_3||Y_{23}| \cos(\theta_{23} - \delta_2 + \delta_3)$$

$= (1.05)\ (22.36068)\ \cos(116.6 - 0 + 0) + 2(1)\ (58.13777)\ \cos(-63.4)$
$+ (1.04)\ (35.77709)\ \cos(116.6 - 0 + 0) = 24.890$

$$\frac{\partial P_3}{\partial \delta_2} = -|V_3||V_2||Y_{32}|\ \sin(\theta_{32} - \delta_3 + \delta_2)$$
$= -(1.04)\ (1)\ (35.77709)\ \sin(116.6 - 0 + 0) = -33.2698$

$$\frac{\partial P_3}{\partial \delta_3} = |V_3||V_1||Y_{31}|\ \sin(\theta_{31} - \delta_3 + \delta_1) + |V_3||V_2||Y_{32}|\ \sin(\theta_{32} - \delta_3 + \delta_2)$$
$= (1.04)\ (1.05)\ (31.62278)\ \sin(108.4 - 0 + 0)$
$+ (1.04)\ (1)\ (35.77709)\ \sin(116.6 - 0 + 0) = 66.0365$

$$\frac{\partial P_3}{\partial |V_2|} = |V_3||Y_{32}|\ \cos(\theta_{32} - \delta_3 + \delta_2)$$
$= (1.04)\ (35.77709)\ \cos(116.6 - 0 + 0) = -16.663$

$$\frac{\partial Q_2}{\partial \delta_2} = |V_2||V_1||Y_{21}|\ \cos(\theta_{21} - \delta_2 + \delta_1) + |V_2||V_3||Y_{23}|\ \cos(\theta_{23} - \delta_2 + \delta_3)$$
$= (1)\ (1.05)\ (22.36068)\ \cos(116.6 - 0 + 0)$
$+ (1)\ (1.04)\ (35.77709)\ \cos(116.6 - 0 + 0) = -27.1731$

$$\frac{\partial Q_2}{\partial \delta_3} = -|V_2||V_3||Y_{23}|\ \cos(\theta_{23} - \delta_2 + \delta_3)$$
$= -(1)\ (1.04)\ (35.77709)\ \cos(116.6 - 0 + 0) = 16.663$

$$\frac{\partial Q_2}{\partial |V_2|} = -|V_1||Y_{21}|\ \sin(\theta_{21} - \delta_2 + \delta_1) - 2|V_2||Y_{22}|\ \sin(\theta_{22})$$
$- |V_3||Y_{23}|\ \sin(\theta_{23} - \delta_2 + \delta_3)$
$= -(1.05)\ (22.36068)\ \sin(116.6 - 0 + 0) - 2(1)\ (58.13777)\ \sin(-63.4)$
$- (1.04)\ (35.77709)\ \sin(116.6 - 0 + 0) = 49.707$

$$\begin{bmatrix} -2.8586 \\ 1.43846 \\ -0.22 \end{bmatrix} = \begin{bmatrix} 54.2634 & -33.2698 & 24.890 \\ -33.2698 & 66.0365 & -16.663 \\ -27.1731 & 16.663 & 49.707 \end{bmatrix} \begin{bmatrix} \Delta\delta_2^{(0)} \\ \Delta\delta_3^{(0)} \\ \Delta|V_2|^{(0)} \end{bmatrix}$$

$$\begin{bmatrix} \Delta\delta_2^{(0)} \\ \Delta\delta_3^{(0)} \\ \Delta|V_2|^{(0)} \end{bmatrix} = \begin{bmatrix} 54.2634 & -33.2698 & 24.890 \\ -33.2698 & 66.0365 & -16.663 \\ -27.1731 & 16.663 & 49.707 \end{bmatrix}^{-1} \begin{bmatrix} -2.8586 \\ 1.43846 \\ -0.22 \end{bmatrix}$$

$$\begin{bmatrix} \Delta\delta_2^{(0)} \\ \Delta\delta_3^{(0)} \\ \Delta|V_2|^{(0)} \end{bmatrix} = \begin{bmatrix} 0.0231 & 0.0134 & -0.0071 \\ 0.0137 & 0.0219 & 0.0005 \\ 0.0081 & 0.0000 & 0.0161 \end{bmatrix} \begin{bmatrix} -2.8586 \\ 1.43846 \\ -0.22 \end{bmatrix}$$

$$\begin{bmatrix} \Delta\delta_2^{(0)} \\ \Delta\delta_3^{(0)} \\ \Delta|V_2|^{(0)} \end{bmatrix} = \begin{bmatrix} -0.0452 \\ -0.0077 \\ -0.0266 \end{bmatrix}$$

New bus voltages and angles in the first iteration are

$$\delta_i^{[k+1]} = \delta_i^{[k]} + \Delta\delta_i^{[k]}$$

$$\delta_2^{[1]} = \delta_2^{[0]} + \Delta\delta_2^{[0]} = 0 + (-0.0452) = -0.0452$$

$$\delta_3^{[1]} = \delta_3^{[0]} + \Delta\delta_3^{[0]} = 0 + (-0.0077) = -0.0077$$

$$|V_i^{[k+1]}| = |V_i^{[k]}| + \Delta|V_i^{[k]}|$$

$$|V_2^{[1]}| = |V_2^{[0]}| + \Delta|V_2^{[0]}| = 1 + (-0.0266) = 0.9734$$

(b) Verify the result using Power World Simulator (PWS): The one line diagram of a simple bus system is drawn in PWS, which is shown in Figure 2.15.

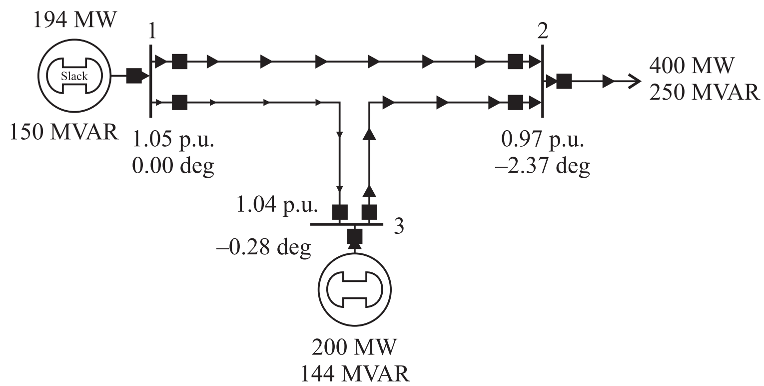


Figure 2.15 One line diagram of a simple three-bus system.

The first step is the formation of $[Y_{\text{bus}}]$ using the inspection method. The calculated $[Y_{\text{bus}}]$ values are given in Figure 2.16. Since the given problem is a three-bus system, the size of $[Y_{\text{bus}}]$ is 3×3 matrix.

	Number	Name	Bus 1	Bus 2	Bus 3
1	1	1	20.00 - j50.00	-10.00 + j20.00	-10.00 + j30.00
2	2	2	-10.00 + j20.00	26.00 - j52.00	-16.00 + j32.00
3	3	3	-10.00 + j30.00	-16.00 + j32.00	26.00 - j62.00

Figure 2.16 Y_{bus} result.

Newton–Raphson method

This method is executed by pressing the icon *Newton–Raphson power flow* available in *tools ribbon*. Before executing this method, the number of iterations is to be fixed as 1 in *simulator options ribbon*. The Jacobian values and power flow results are given in Figure 2.17 and Figure 2.18 for the 1st iteration.

	Number	Name	Jacobian Equation	Angle Bus 2	Angle Bus 3	Volt Mag Bus 2	Volt Mag Bus 3
1	2	2	Real Power	51.71	-31.76	21.24	-16.80
2	3	3	Real Power	-33.03	65.71	-15.32	28.99
3	2	2	Reactive Power	-28.66	17.47	48.23	-30.53
4	3	3	Voltage Magnitude				1.00

Figure 2.17 Jacobian values.

```
Bus Flows
BUS      1 1          138.0   MW     Mvar      MVA   % 1.0500    0.00   1 1
 GENERATOR 1              213.54  136.29R  253.3
 TO       2 2          1  177.35  113.36   210.5   0
 TO       3 3          1   36.18   22.94    42.8   0
 **** Mismatch ****       213.54  136.29
BUS      2 2          138.0   MW     Mvar      MVA   % 0.9741   -2.70   1 1
 LOAD 1                   400.00  250.00   471.7
 TO       1 1          1 -169.32  -97.28   195.3   0
 TO       3 3          1 -229.08 -139.14   268.0   0
 **** Mismatch ****        -1.60  -13.58
BUS      3 3          138.0   MW     Mvar      MVA   % 1.0400   -0.45   1 1
 GENERATOR 1              200.00  135.63R  241.6
 TO       1 1          1  -36.01  -22.44    42.4   0
 TO       2 2          1  238.54  158.07   286.2   0
 **** Mismatch ****       197.47  135.62
```

Figure 2.18 Power flow results and voltages—1st iteration.

The details of convergence are shown in Figure 2.19. The mismatches of real powers and reactive power for each iteration are also clearly indicated. This method takes 2 iterations to converge power flows. The converged values of Jacobian and Power flow results are given in Figure 2.20 and Figure 2.21.

Message Log: book_sample.PWB

Starting Solution using Rectangular Newton-Raphson

Number: 0 Max P: 286.000 at bus 2 Max Q: 22.000 at bus 2
Number: 1 Max P: 2.530 at bus 3 Max Q: 13.578 at bus 2
Number: 2 Max P: 0.004 at bus 2 Max Q: 0.001 at bus 2

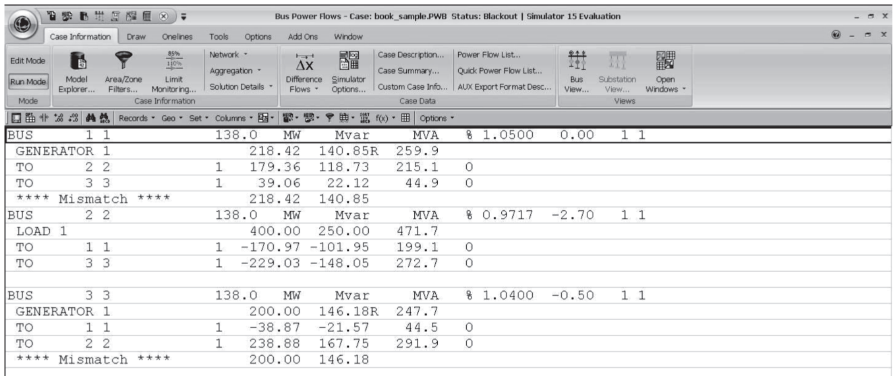
Finished voltage control loop iteration: 1

Solution Finished in 0.000 Seconds
Simulation: Successful Power Flow Solution

Figure 2.19 Details of convergence.

	Number	Name	Jacobian Equation	Angle Bus 2	Angle Bus 3	Volt Mag Bus 2	Volt Mag Bus 3
1	2	2	Real Power	51.60	-31.69	21.15	-16.73
2	3	3	Real Power	-32.93	65.60	-15.35	28.96
3	2	2	Reactive Power	-28.55	17.40	47.95	-30.47
4	3	3	Voltage Magnitude				1.00

Figure 2.20 Jacobian values.



BUS 1 1		138.0	MW	Mvar	MVA	% 1.0500	0.00	1 1
GENERATOR 1			218.42	140.85R	259.9			
TO 2 2	1		179.36	118.73	215.1	0		
TO 3 3	1		39.06	22.12	44.9	0		
**** Mismatch ****			218.42	140.85				
BUS 2 2		138.0	MW	Mvar	MVA	% 0.9717	-2.70	1 1
LOAD 1			400.00	250.00	471.7			
TO 1 1	1		-170.97	-101.95	199.1	0		
TO 3 3	1		-229.03	-148.05	272.7	0		
BUS 3 3		138.0	MW	Mvar	MVA	% 1.0400	-0.50	1 1
GENERATOR 1			200.00	146.18R	247.7			
TO 1 1	1		-38.87	-21.57	44.5	0		
TO 2 2	1		238.88	167.75	291.9	0		
**** Mismatch ****			200.00	146.18				

Figure 2.21 Converged power flow results and voltages.

PSS/E

The same problem is taken and drawn in PSS/E software and it is given in Figure 2.22.

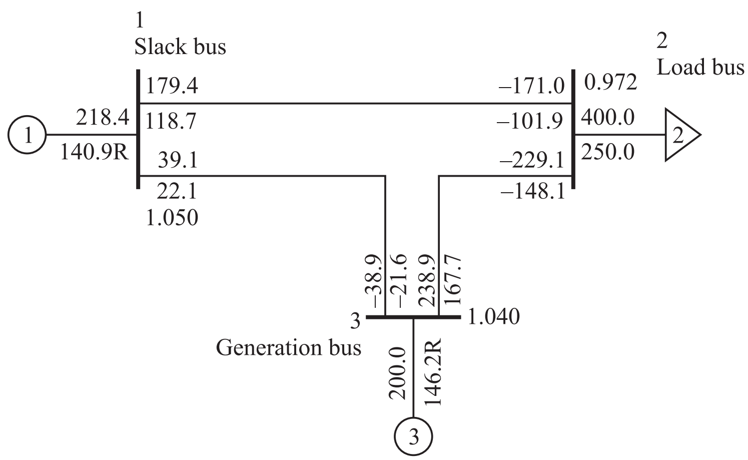


Figure 2.22 One line diagram of a simple three-bus system.

Once the data are entered in the software it can be executed by the above three power flow methods. Figure 2.23 shows the converged results obtained by the Gauss–Seidel method. This window is generated from ***bus based report***.

The Newton–Raphson method is executed and the power flow results are shown in Figure 2.23.

2.7 Fast Decoupled Load Flow Method

The Fast Decoupled Load Flow (FDLF) method is one of the improved methods, which was based on the simplification of the Newton–Raphson method and reported by Stott and Alsac in 1974. This method due to its simplifications

```
           PTI INTERACTIVE POWER          Mon, JUN 27  2011     11:50    %MVA FOR TRANSFORMERS
         SYSTEM SIMULATOR--PSS®E                              RATING     %I     FOR NON-TRANSFORMER BRANCHES
                                                               SET A

BUS     1 SLACK BUS    CKT    MW      MVAR      MVA   %  1.050PU   0.00  X---LOSSES---X  X---AREA----X  X---ZONE---X  1
  FROM GENERATION           218.4    140.8R    259.9 260      kV          MW     MVAR       1              1
  TO    2 LOAD BUS      1   179.3    118.7     215.1                      8.39   16.78      1              1
  TO    3 GEN BUS       1    39.1     22.1      44.9                      0.18    0.55      1              1

BUS     2 LOAD BUS     CKT    MW      MVAR      MVA   % 0.9717PU  –2.70  X---LOSSES---X  X---AREA----X  X---ZONE---X  2
                                                                  kV      MW     MVAR       1              1
  TO LOAD-PQ                400.0    250.0     471.7
  TO    1 SLACK BUS     1  –171.0   –101.9     199.0                      8.39   16.78      1              1
  TO    3 GEN BUS       1  –229.0   –148.0     272.7                      9.85   19.69      1              1

BUS     3 GEN BUS      CKT    MW      MVAR      MVA   % 1.0400PU  –0.50  X---LOSSES---X  X---AREA----X  X---ZONE---X  3
  FROM GENERATION           200.0    146.2R    247.7 248      kV          MW     MVAR       1              1
  TO    1 SLACK BUS     1   –38.9    –21.6      44.5                      0.18    0.55      1              1
  TO    2 LOAD BUS      1   238.9    167.7     291.9                      9.85   19.69      1              1
                                                                       Report
```

Figure 2.23 Converged results using Newton–Raphson method.

of calculations, fast convergence and reliable results became the most widely used method in load flow analysis.

However, FDLF for some cases, where high *R*/*X* ratios or heavy loading (low voltage) at some buses are present, does not converge well. For these cases, many efforts and developments have been made to overcome these convergence obstacles. Some of them targeted the convergence of systems with high *R*/*X* ratios, others those with low voltage buses. However, one of the most recent developments is a Robust Fast Decoupled Load Flow developed by Wang and Li; it is based on heuristic justification and general voltage normalization methods and solves both high *R*/*X* ratios and low bus voltage problem simultaneously.

This method exploits the property of the power system wherein real power flow-voltage angle ($P = (V_1V_2/X_1)\sin\delta$) and reactive power flow-voltage magnitude are loosely ($Q = (V_1V_2/X)\cos\delta - (V_2^2/X)$) coupled.

As the FDLF is derived from the Newton–Raphson method, we will start from the matrix representation of Newton–Raphson and apply some simplifications and approximations to reach the equations of the FDLF.

The matrix representation of the Newton–Raphson method is:

$$\begin{bmatrix}\Delta P\\ \Delta Q\end{bmatrix}=\begin{bmatrix}J_1 & J_2\\ J_3 & J_4\end{bmatrix}\begin{bmatrix}\Delta\delta\\ \Delta|V|\end{bmatrix} \tag{2.57}$$

Elements of Jacobian matrix $\mathbf{J}_1$

(i) the diagonal elements are

$$\frac{\partial P_i}{\partial \delta_i}=\sum_{j\neq i}|V_i||V_j||Y_{ij}|\sin(\theta_{ij}-\delta_i+\delta_j) \tag{2.58}$$

(ii) the off-diagonal elements are

$$\frac{\partial P_i}{\partial \delta_j}=-|V_i||V_j||Y_{ij}|\sin(\theta_{ij}-\delta_i+\delta_j) \qquad j\neq i \tag{2.59}$$

Elements of Jacobian matrix $\mathbf{J}_2$

(i) the diagonal elements are

$$\frac{\partial P_i}{\partial |V_i|}=2|V_i||Y_{ii}|\cos\theta_{ii}+\sum_{j\neq i}|V_j||Y_{ij}|\cos(\theta_{ij}-\delta_i+\delta_j) \tag{2.60}$$

(ii) the off-diagonal elements are

$$\frac{\partial P_i}{\partial |V_j|}=|V_i||Y_{ij}|\cos(\theta_{ij}-\delta_i+\delta_j) \qquad j\neq i \tag{2.61}$$

Elements of Jacobian matrix $\mathbf{J}_3$

(i) the diagonal elements are

$$\frac{\partial Q_i}{\partial \delta_i}=\sum_{j\neq i}|V_i||V_j||Y_{ij}|\cos(\theta_{ij}-\delta_i+\delta_j) \tag{2.62}$$

(ii) the off-diagonal elements are

$$\frac{\partial Q_i}{\partial \delta_j} = -|V_i||V_j||Y_{ij}|\cos(\theta_{ij} - \delta_i + \delta_j) \qquad j \neq i \tag{2.63}$$

Elements of Jacobian matrix $\mathbf{J}_4$

(i) the diagonal elements are

$$\frac{\partial Q_i}{\partial |V_i|} = -2|V_i||Y_{ii}|\sin\theta_{ii} - \sum_{j\neq i}|V_j||Y_{ij}|\sin(\theta_{ij} - \delta_i + \delta_j) \tag{2.64}$$

(ii) the off-diagonal elements are

$$\frac{\partial Q_i}{\partial |V_j|} = -|V_i||Y_{ij}|\sin(\theta_{ij} - \delta_i + \delta_j) \qquad j \neq i \tag{2.65}$$

Now, for typical power system branches:

$$X/R \gg 1 \text{ and } \theta_{ij} < 20° \tag{2.66}$$

These two approximations will cause a weak coupling between ΔP and ΔV, and between ΔQ and $\Delta\delta$, hence $\mathbf{J}_2$ and $\mathbf{J}_3$ entries of the initial matrix of equation (2.57) can be ignored leading to the following decoupled equations:

$$\begin{bmatrix} \Delta P \\ \Delta Q \end{bmatrix} = \begin{bmatrix} J_1 & 0 \\ 0 & J_4 \end{bmatrix} \begin{bmatrix} \Delta\delta \\ \Delta|V| \end{bmatrix} \tag{2.67}$$

$$[\Delta P] = [J_1][\Delta\delta] = \left[\frac{\partial P}{\partial \delta}\right][\Delta\delta] \tag{2.68}$$

$$[\Delta Q] = [J_4][\Delta|V|] = \left[\frac{\partial Q}{\partial |V|}\right][\Delta|V|] \tag{2.69}$$

Equations (2.68) and (2.69) show that the matrix equations are separated into two decoupled equations requiring considerably less time to solve compared to the time required for the solution of Eq. (2.57).

Furthermore, considerable simplifications can be made to eliminate the need for recalculating $\mathbf{J}_1$ and $\mathbf{J}_4$ during iteration.

The elements of Jacobian matrix $\mathbf{J}_1$ are as follows.

The diagonal elements are

$$\frac{\partial P_i}{\partial \delta_j} = \sum_{j=1}^{n}|V_i||V_j||Y_{ij}|\sin(\theta_{ij} - \delta_i + \delta_j) - |V_i|^2\,|Y_{ii}|\sin(\theta_{ii})$$

$$\frac{\partial P_i}{\partial \delta_j} = -Q_i - |V_i|^2|Y_{ii}|\,\sin(\theta_{ii})$$

$$\frac{\partial P_i}{\partial \delta_j} = -Q_i - |V_i|^2 B_{ii}$$

Now, the diagonal elements of $\mathbf{J}_1$ can be written as

$$\frac{\partial P_i}{\partial \delta_i} = -Q_i - |V_i|^2 B_{ii} \tag{2.70}$$

where $B_{ii} = |Y_{ii}| \sin\theta_{ii}$ is the imaginary part of the diagonal elements of the bus admittance matrix Y_{bus}.

Further simplifications can be applied to Eq. (2.70), by considering

$$B_{ii} \gg Q_i \text{ and } |V_i|^2 \approx |V_i|$$

$$\frac{\partial P_i}{\partial \delta_i} = -|V_i| B_{ii} \tag{2.71}$$

Also, as under normal operating conditions $\delta_j - \delta_i$ is quite small, therefore $\theta_{ij} - \delta_i + \delta_j \approx \theta_{ij}$ and $|V_j| \approx 1$.

The off-diagonal elements of $\mathbf{J}_1$ can be written as

$$\frac{\partial P_i}{\partial \delta_j} = -|V_i||V_j||Y_{ij}| \sin(\theta_{ij} - \delta_i + \delta_j) \qquad \therefore |V_j| \simeq 1$$

$$= -|V_i||Y_{ij}| \sin(\theta_{ij})$$

$$\frac{\partial P_i}{\partial \delta_j} = -|V_i| B_{ij} \tag{2.72}$$

Similarly, the diagonal elements of $\mathbf{J}_4$ may be written as

$$\frac{\partial Q_i}{\partial |V_i|} = -|V_i||Y_{ii}| \sin\theta_{ii} - \sum_{j=1}^{n} |V_j||Y_{ij}| \sin(\theta_{ij} - \delta_i + \delta_j)$$

Multiplying the above equation by $|V_i|$, we get

$$|V_i| \times \frac{\partial Q_i}{\partial |V_i|} = -|V_i|^2 |Y_{ii}| \sin\theta_{ii} - \sum_{j=1}^{n} |V_i||V_j||Y_{ij}| \sin(\theta_{ij} - \delta_i + \delta_j) = -|V_i|^2 B_{ii} + Q_i$$

Again, since $B_{ii} \gg Q_i$, Q_i may be neglected

$$\frac{\partial Q_i}{\partial |V_i|} = -|V_i| B_{ii} \tag{2.73}$$

The off-diagonal elements of $\mathbf{J}_4$ are

$$\frac{\partial Q_i}{\partial |V_j|} = -|V_i||Y_{ij}| \sin(\theta_{ij} - \delta_i + \delta_j)$$

Again assume $\theta_{ij} - \delta_i + \delta_j \approx \theta_{ij}$

$$\frac{\partial Q_i}{\partial |V_j|} = -|V_i||Y_{ij}| \sin\theta_{ij}$$

$$\frac{\partial Q_i}{\partial |V_j|} = -|V_i| B_{ij} \tag{2.74}$$

Applying these assumptions to Eqs. (2.68) and (2.69), we get

$$\frac{\partial P_i}{\partial \delta_i} = -|V_i| B_{ii} \quad \text{or} \quad \frac{\Delta P_i}{\Delta \delta_i} = -|V_i| B_{ii}$$

$$\frac{\Delta P_i}{|V_i|} = -B_{ii} \Delta \delta_i$$

$$\frac{\Delta P}{|V_i|} = -B' \Delta \delta_i \tag{2.75}$$

Similarly,

$$\frac{\partial Q_i}{\partial |V_i|} = -|V_i| B_{ii} \quad \text{or} \quad \frac{\Delta Q_i}{\Delta |V_i|} = -|V_i| B_{ii}$$

$$\frac{\Delta Q_i}{\Delta |V_i|} = -B_{ii} \Delta |V_i|$$

$$\frac{\Delta Q}{|V_i|} = -B'' \Delta |V_i| \tag{2.76}$$

where, B' and B'' are the imaginary part of the bus admittance matrix Y_{bus}, such that B' contains all buses admittance except those related to the slack bus, and B'' is B' deprived from all voltage controlled buses related admittances.

Finally, all these approximations and simplifications lead to the following successive voltage magnitude and voltage angle updating equations.

$$\Delta \delta = -[B']^{-1} \frac{\Delta P}{|V|} \tag{2.77}$$

$$\Delta V = -[B'']^{-1} \frac{\Delta Q}{|V|} \tag{2.78}$$

FDLF technique is very useful in contingency analysis where numerous outages are to be simulated or a load flow solution is required for online control.

The algorithm written according to the equations derived in the previous section is as follows:

Step 1: Create the bus admittance matrix $[Y_{\text{bus}}]$.

Step 2: Detect all kinds and numbers of buses and setting all bus voltages to an initial value of 1 p.u., all voltage angles to 0, and the iteration counter *iter* to 0.

Step 3: Create the matrices B' and B'' according to Eqs. (2.75) and (2.76).

Step 4: *If* max (ΔP, ΔQ) ≤ accuracy

$$\Delta P_i^{[k]} = P_{i,\text{sch}} - P_i^{[k]}$$

$$\Delta Q_i = Q_{i,\text{sch}} - Q_i^{[k]}$$

then go to Step 6

else

(i) Calculate $\mathbf{J}_1$ and $\mathbf{J}_4$ elements of Eqs. (2.71), (2.72), (2.73) and (2.74).

$$\frac{\partial P_i}{\partial \delta_i} = -|V_i|B_{ii} \qquad \frac{\partial P_i}{\partial \delta_j} = -|V_i|B_{ij}$$
$$\frac{\partial Q_i}{\partial |V_i|} = -|V_i|B_{ii} \qquad \frac{\partial Q_i}{\partial |V_j|} = -|V_i|B_{ij}$$

(ii) Calculate the real and reactive powers at each bus, and check if MVAR of generator buses are within the limits, otherwise update the voltage magnitude at these buses by ± 2 %.

If $Q_{i,\min} < Q_i < Q_{i,\max}$, calculate $P_i^{(k)}$

If $Q_i^{[k]} > Q_{i,\max}$, $\quad Q_{i,\text{sch}} = Q_{i,\max}$

If $Q_i^{[k]} < Q_{i,\min}$, $\quad Q_{i,\text{sch}} = Q_{i,\min}$

The *PV* bus will act as *PQ* bus.

(iii) Calculate the power residuals, ΔP and ΔQ.

$$\Delta P_i^{[k]} = P_{i,\text{sch}} - P_i^{[k]}$$
$$\Delta Q_i^{[k]} = Q_{i,\text{sch}} - Q_i^{[k]}$$

(iv) Calculate the bus voltage and voltage angle updates ΔV and $\Delta\delta$.

$$[\Delta\delta_i]^{(k)} = -[B']^{-1} \frac{\Delta P_i^{[k]}}{|V_i|}$$

$$[\Delta V_i]^{(k)} = -[B'']^{-1} \frac{\Delta Q_i^{[k]}}{|V_i|}$$

(v) Update the voltage magnitude V and the voltage angle δ at each bus.

$$\delta_i^{[k+1]} = \delta_i^{[k]} + \Delta\delta_i^{[k]}$$
$$|V_i^{[k+1]}| = |V_i^{[k]}| + \Delta|V_i^{[k]}|$$

(vi) Increment of the iteration counter *iter* = *iter* + 1

Step 5: If iter ≤ maximum number of iteration

$$|\Delta P_i^{[k]}| \le \varepsilon$$
$$|\Delta Q_i^{[k]}| \le \varepsilon$$

then go to Step 4

else print out ‘Solution did not converge’ and go to Step 6.

Step 6: Print out of the power flow solution, computation and display of the line flow and losses.

This completes the load flow study. Finally, in Figure 2.24 all the computational steps are summarized in the detailed flow chart.

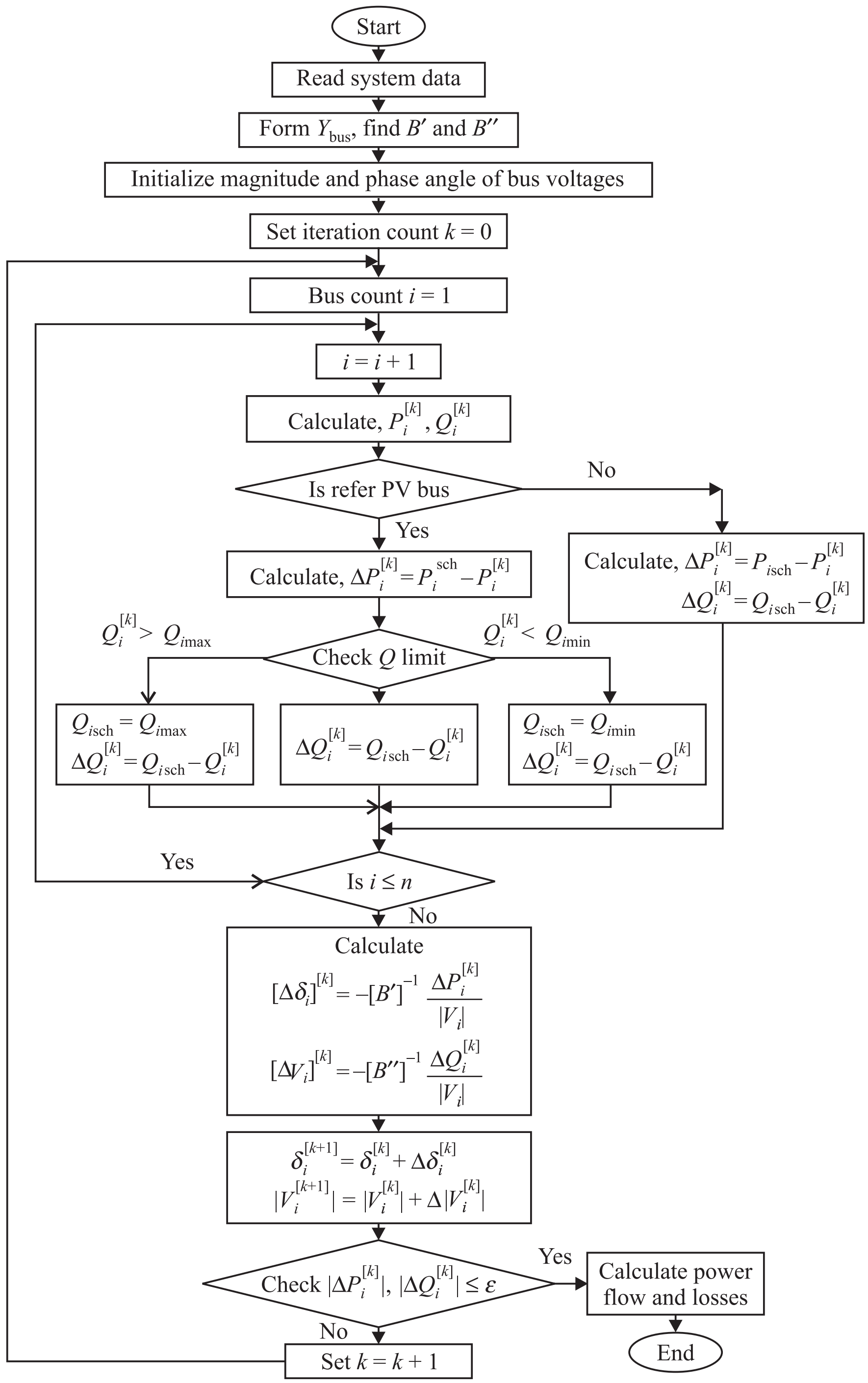


Figure 2.24 Flow chart for FDLF method.

EXAMPLE 2.7 Figure 2.25 shows the one line diagram of a simple three-bus system with generation at bus 1. The magnitude of voltage at bus 1 is adjusted to 1.05 p.u. The scheduled loads at buses 2 and 3 are given in the diagram. Line impedances are marked as *n* p.u. on a 100 MVA base and the line charging susceptances are neglected.

(a) Using the fast decoupled load flow method, determine the phasor values of the voltages at the load buses 2 and 3(*PQ* bus) accurate to decimal places.
(b) Verify the result with Power World Simulator and PSS/E.

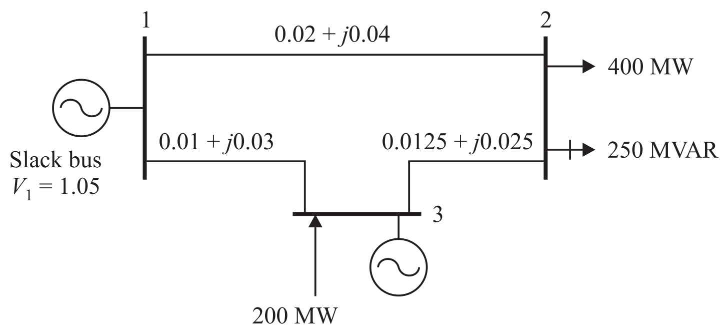


Figure 2.25 One line diagram of a simple three-bus system.

Solution: (a) Form the Y_{bus}

$$y_{12} = \frac{1}{z_{12}} = \frac{1}{0.02 + j0.04} = 10 - j20$$

$$y_{13} = \frac{1}{z_{13}} = \frac{1}{0.01 + j0.03} = 10 - j30$$

$$y_{23} = \frac{1}{z_{23}} = \frac{1}{0.0125 + j0.025} = 16 - j32$$

$$Y_{11} = y_{12} + y_{13} = (10 - j20) + (10 - j30) = 20 - j50$$

$$Y_{12} = Y_{21} = -y_{12} = -(10 - j20) = -10 + j20$$

$$Y_{13} = Y_{31} = -y_{13} = -(10 - j30) = -10 + j30$$

$$Y_{22} = y_{21} + y_{23} = (10 - j20) + (16 - j32) = 26 - j52$$

$$Y_{23} = Y_{32} = -y_{23} = -(16 - j32) = -16 + j32$$

$$Y_{33} = y_{31} + y_{32} = (10 - j30) + (16 - j32) = 26 - j62$$

$$Y_{\text{bus}} = \begin{bmatrix} 20 - j50 & -10 + j20 & -10 + j30 \\ -10 + j20 & 26 - j52 & -16 + j32 \\ -10 + j30 & -16 + j32 & 26 - j62 \end{bmatrix}$$

Bus 1 is slack bus and the corresponding bus susceptance matrix for evaluation of phase angle $\Delta\delta_2$ and $\Delta\delta_3$ is

$$B' = \begin{bmatrix} -52 & 32 \\ 32 & -62 \end{bmatrix}$$

The inverse of the above matrix is

$$[B']^{-1} = \begin{bmatrix} -0.028182 & -0.014545 \\ -0.014545 & -0.023636 \end{bmatrix}$$

Initialize magnitude and angle of bus voltage

$$|V_1| = 1.05,\ \delta_1 = 0.0 \text{ rad}$$

$$|V_2|^{(0)} = 1,\ \delta_2^{(0)} = 0.0 \text{ rad}$$

$$|V_3|^{(0)} = 1.04,\ \delta_3^{(0)} = 0.0 \text{ rad}$$

In the matrix form

$$\begin{bmatrix} \delta_1^{(0)} \\ V_1^{(0)} \end{bmatrix} = \begin{bmatrix} 0 \\ 1.05 \end{bmatrix};\ \begin{bmatrix} \delta_2^{(0)} \\ V_2^{(0)} \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \end{bmatrix};\ \begin{bmatrix} \delta_3^{(0)} \\ V_3^{(0)} \end{bmatrix} = \begin{bmatrix} 0 \\ 1.04 \end{bmatrix}$$

Scheduled powers are

at bus 2, $P_{2,\text{sch}} = P_{G2} - P_{D2} = 0 - \dfrac{400}{100} = -4$ p.u.

$$Q_{2,\text{sch}} = Q_{G2} - Q_{D2} = 0 - \frac{250}{100} = -2.5 \text{ p.u.}$$

at bus 3, $P_{3,\text{sch}} = P_{G3} - P_{D3} = \dfrac{200}{100} - 0 = 2$ p.u.

The real power at buses 2 and 3 and reactive power at bus 2 are

$$P_2 = |V_2||V_1||Y_{21}| \cos(\theta_{21} - \delta_2 + \delta_1) + |V_2^2||Y_{22}| \cos\theta_{22} + |V_2||V_3||Y_{23}| \cos(\theta_{23} - \delta_2 + \delta_3)$$

$$P_2 = (1)\ (1.05)\ (22.36068) \cos(116.6 - 0 + 0) + (1)^2 (58.13777) \cos(-63.4) + (1)\ (1.04)\ (35.77709) \cos(116.6 - 0 + 0) = -1.1414$$

$$P_3 = |V_3||V_1||Y_{31}| \cos(\theta_{31} - \delta_3 + \delta_1) + |V_3||V_2||Y_{32}| \cos(\theta_{32} - \delta_3 + \delta_2) + |V_3^2||Y_{33}| \cos\theta_{33}$$

$$P_3 = (1.04)\ (1.05)\ (31.62278) \cos(108.4 - 0 + 0) + (1.04)(1)(35.77709) \cos(116.6 - 0 + 0) + (1.04)^2 (67.23095) \cos(-67.2) = 0.5616$$

$$Q_2 = -|V_2||V_1||Y_{21}| \sin(\theta_{21} - \delta_2 + \delta_1) - |V_2^2||Y_{22}| \sin\theta_{22} - |V_2||V_3||Y_{23}| \sin(\theta_{23} - \delta_2 + \delta_3)$$

$$Q_2 = -(1)\ (1.05)\ (22.36068) \sin(116.6 - 0 + 0) - (1)^2 (58.13777) \sin(-63.4) - (1)\ (1.04)\ (35.77709) \sin(116.6 - 0 + 0) = -2.28$$

Difference in scheduled to calculated power

$$\Delta P_2^{[0]} = P_{2,\text{sch}} - P_{2,\text{calc}}^{(0)} = -4 - (-1.1414) = -2.8586$$

$$\Delta P_3^{[0]} = P_{3,\text{sch}} - P_{3,\text{calc}}^{(0)} = 2 - (0.5616) = 1.43846$$

$$\Delta Q_2^{[0]} = Q_{2,\text{sch}} - Q_{2,\text{calc}}^{(0)} = -2.5 - (-2.28) = -0.22$$

The FDLF algorithm given by Eq. (2.77) becomes

$$[\Delta \delta_i]^{(k)} = -[B']^{-1} \frac{\Delta P_i^{[k]}}{|V_i|}$$

$$\begin{bmatrix} \delta_2^{(0)} \\ \delta_3^{(0)} \end{bmatrix} = -[B']^{-1} \begin{bmatrix} \dfrac{\Delta P_2^{[0]}}{|V_2|} \\ \dfrac{\Delta P_3^{[0]}}{|V_3|} \end{bmatrix}$$

$$\begin{bmatrix} \delta_2^{(0)} \\ \delta_3^{(0)} \end{bmatrix} = -\begin{bmatrix} -0.028182 & -0.014545 \\ -0.014545 & -0.023636 \end{bmatrix} \begin{bmatrix} \dfrac{-2.8586}{1.0} \\ \dfrac{1.43846}{1.04} \end{bmatrix}$$

$$= \begin{bmatrix} -0.028182 & -0.014545 \\ -0.014545 & -0.023636 \end{bmatrix} \begin{bmatrix} -2.8586 \\ 1.3831 \end{bmatrix}$$

$$= \begin{bmatrix} -0.060483 \\ -0.008909 \end{bmatrix}$$

Since bus 3 is a regulated bus, the corresponding row and column of B' are eliminated and we get

$$B'' = [-52]$$

$$[B'']^{-1} = \frac{-1}{52} = -0.01923$$

$$[\Delta V_i]^{(k)} = -[B'']^{-1} \frac{\Delta Q_i^{[k]}}{|V_i|}$$

$$[\Delta V_2]^{(0)} = -[B'']^{-1} \frac{\Delta Q_0^{[0]}}{|V_2|}$$

$$[\Delta V_2]^{(0)} = -(-0.01923) \left[\frac{-0.22}{1.0} \right] = -0.0042308$$

The new bus voltages and the angles in the first iteration are

$$\delta_i^{[k+1]} = \delta_i^{[k]} + \Delta \delta_i^{[k]}$$

$$\delta_2^{[1]} = \delta_2^{[0]} + \Delta \delta_2^{[0]} = 0 + (-0.060483) = -0.060483$$

$$\delta_3^{[1]} = \delta_3^{[0]} + \Delta \delta_3^{[0]} = 0 + (-0.008909) = -0.008909$$

$$|V_i^{[k+1]}| = |V_i^{[k]}| + \Delta |V_i^{[k]}|$$

$$|V_2^{[1]}| = |V_2^{[0]}| + \Delta|V_2^{[0]}| = 1 + (-0.0042308) = 0.99577$$

(b) Verify the result using Power World Simulator: The one line diagram of a simple bus system drawn in PWS is shown in Figure 2.26.

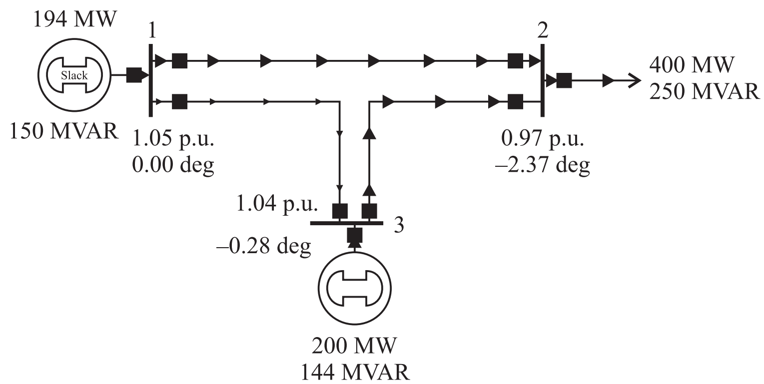


Figure 2.26 One line diagram of a simple three-bus system.

The first step is the formation of $[Y_{bus}]$ using the inspection method. The calculated $[Y_{bus}]$ values are given in Figure 2.27. Since the given problem is a three-bus system, the size of $[Y_{bus}]$ is 3 × 3 matrix.

Y Bus (Bus Admittance Matrix)

Records Geo Set Columns f(x)

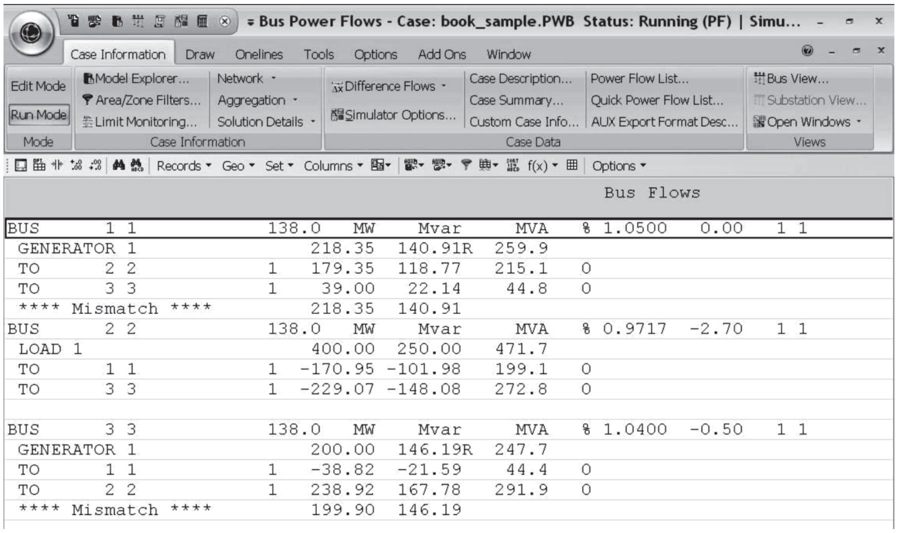
Filter Advanced Bus Find... Remove

	Number	Name	Bus 1	Bus 2	Bus 3
1	1	1	20.00 - j50.00	-10.00 + j20.00	-10.00 + j30.00
2	2	2	-10.00 + j20.00	26.00 - j52.00	-16.00 + j32.00
3	3	3	-10.00 + j30.00	-16.00 + j32.00	26.00 - j62.00

Figure 2.27 Y_{bus} result.

This method is executed by pressing the icon *fast decoupled* available in *tools ribbon*. Before executing this method, the number of iterations is to be fixed as 1 in *simulator options ribbon*.

The same problem has been executed by the fast decoupled method. The converged results are given in Figure 2.28. The converged power flow results for Gauss–Seidel, Newton–Raphson and fast decoupled are shown in Figures 2.8, 2.21 and 2.28. The power flow results are the same for all methods, but the results are converged quickly by Newton–Raphson method, i.e. by two iterations.



BUS 1 1	138.0	MW	Mvar	MVA	% 1.0500	0.00	1 1
GENERATOR 1		218.35	140.91R	259.9			
TO 2 2	1	179.35	118.77	215.1	0		
TO 3 3	1	39.00	22.14	44.8	0		
**** Mismatch ****		218.35	140.91				
BUS 2 2	138.0	MW	Mvar	MVA	% 0.9717	-2.70	1 1
LOAD 1		400.00	250.00	471.7			
TO 1 1	1	-170.95	-101.98	199.1	0		
TO 3 3	1	-229.07	-148.08	272.8	0		
BUS 3 3	138.0	MW	Mvar	MVA	% 1.0400	-0.50	1 1
GENERATOR 1		200.00	146.19R	247.7			
TO 1 1	1	-38.82	-21.59	44.4	0		
TO 2 2	1	238.92	167.78	291.9	0		
**** Mismatch ****		199.90	146.19				

Figure 2.28 Converged power flow results and voltages.

PSS/E

The same problem is taken and drawn in PSS/E software and it is given in Figure 2.29.

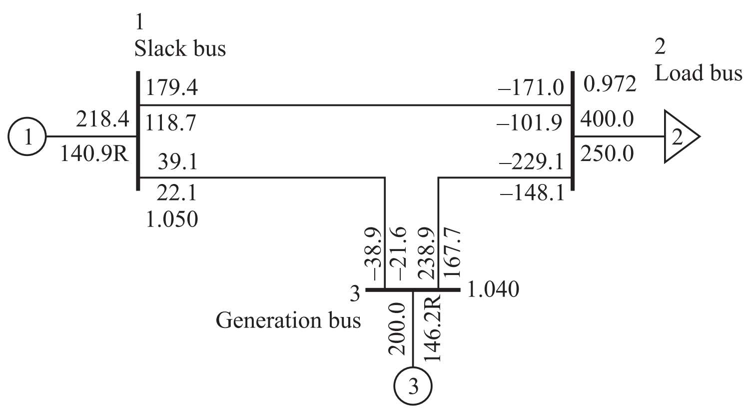


Figure 2.29 One line diagram of a simple three-bus system.

Once the data are entered in the software it can be executed by the above three power flow methods. Figure 2.30 shows the converged results obtained by the fast decoupled method. This window is generated from *bus based report*.

The fast decoupled method is executed and the power flow results are shown in Figure 2.30.

```
             PTI INTERACTIVE POWER          Mon, JUN 27  2011    12:00     %MVA FOR TRANSFORMERS
           SYSTEM SIMULATOR--PSS®E                              RATING    %I     FOR NON-TRANSFORMER BRANCHES
                                                                 SET A

BUS    1 SLACK BUS     CKT    MW      MVAR      MVA   %   1.050PU   0.00   X---LOSSES---X  X---AREA----X  X---ZONE---X  1
  FROM GENERATION            218.4    140.9R   259.9  260      kV           MW     MVAR       1              1
  TO    2 LOAD BUS      1    179.4    118.7    215.1                        8.39   16.79      1              1
  TO    3 GEN BUS       1     39.0     22.1     44.9                        0.18    0.55      1              1

BUS    2 LOAD BUS      CKT    MW      MVAR      MVA   %  0.9717PU  –2.70   X---LOSSES---X  X---AREA----X  X---ZONE---X  2
                                                               kV           MW     MVAR       1              1
  TO LOAD-PQ                 400.0    250.0    471.7
  TO    1 SLACK BUS     1   –171.0   –102.0    199.1                        8.39   16.79      1              1
  TO    3 GEN BUS       1   –229.0   –148.1    272.7                        9.85   19.69      1              1

BUS    3 GEN BUS       CKT    MW      MVAR      MVA   %  1.0400PU  –0.50   X---LOSSES---X  X---AREA----X  X---ZONE---X  3
  FROM GENERATION            200.0    146.2R   247.7  248      kV           MW     MVAR       1              1
  TO    1 SLACK BUS     1    –38.9    –21.6     44.5                        0.18    0.55      1              1
  TO    2 LOAD BUS      1    238.9    167.7    291.9                        9.85   19.69      1              1
```

Figure 2.30 Converged results using fast decoupled method.

2.8 Comparison of the Gauss–Seidel, Newton–Raphson and Fast Decoupled Methods of Load Flow Study

S.No.	*Gauss–Seidel*	*Newton–Raphson*	*Fast decoupled*
1.	Requires a large number of iterations to reach convergence.	Requires a less number of iterations to reach convergence.	Requires a more number of iterations than Newton–Raphson method.
2.	Computation time per iteration is less.	Computation time per iteration is more.	Computation time per iteration is less.
3.	It has linear convergence characteristics.	It has quadratic convergence characteristics.	—
4.	The number of iterations required for convergence increases with the size of the system.	The number of iterations are independent of the size of the system.	The number of iterations does not depend on the size of the system.
5.	Less memory required.	More memory required.	Less memory required than Newton–Raphson method.

Review Questions

Part-A

1. What is the power flow study or load flow study?
2. What are the scraps of information that are obtained from the load flow study?
3. What is the need for load flow study?
4. What are the quantities associated with each bus in a system?
5. What are the different types of buses in a power system? Or, how are the buses classified and what are its types?
6. What is the need for slack bus?
7. Why do we go for iterative methods to solve the load flow problems?
8. What are the methods mainly used for the solution of load flow study?
9. What do you mean by a flat voltage start?
10. Discuss the effect of acceleration factor in load flow study.
11. When is the generator bus treated as load bus?
12. What are the advantages and disadvantages of Gauss–Seidel method?
13. What are the advantages and disadvantages of Newton–Raphson method?
14. Compare the Gauss–Seidel and the Newton–Raphson methods of load flow study.

Part-B

1. The system data for a load flow solution are given in the following tables. Determine the voltages at the end of first iteration by Gauss–Seidel method. Take $\alpha = 1.6$. Verify the result with Power World Simulator.

Bus code	*R* in p.u.	*X* in p.u.
1–2	0.05	0.15
1–3	0.10	0.30
1–4	0.20	0.40
2–4	0.10	0.30
3–4	0.05	0.15

Bus code	*P*	*Q*	*V*	*Remarks*
1	—	—	1.05	Slack bus
2	0.5	–0.2	—	*PQ* bus
3	–1.0	0.5	—	*PQ* bus
4	0.3	–0.1	—	*PQ* bus

2. The system contains six buses. The bus data, branch data and generator data are given below. The system data are prepared and the power flows are solved by Newton–Raphson method using **Power World Simulator**.

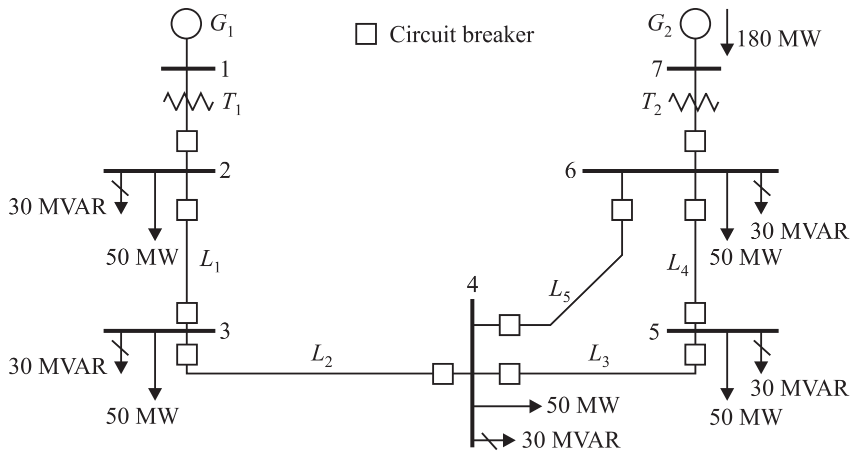
Bus data

Bus	*Type*	*V* p.u.	δ *degree*	P_g MW	Q_g p.u.	P_L p.u.	Q_L p.u.	*Nominal voltage in* kV
1	Swing	1.05	0	—	—	—	—	230
2	Generator	1.05	—	66.368	0	0.0	0.0	230
3	Generator	1.07	—	77.473	—	0.0	0.0	230
4	Load	1.00	—	0	0	70	70	230
5	Load	1.00	—	0	0	70	70	230
6	Load	1.00	—	0	0	70	70	230

Line data

From Bus	*To bus*	*R* p.u.	*X* p.u.	*B* p.u.	*Max. MVA* p.u.
1	2	0.20	0.20	0.02	40
1	4	0.05	0.20	0.02	60
1	5	0.08	0.30	0.03	40
2	3	0.05	0.25	0.03	40
2	4	0.05	0.10	0.01	60
2	5	0.10	0.30	0.02	30
2	6	0.07	0.20	0.025	90
3	5	0.12	0.26	0.025	70
3	6	0.02	0.10	0.01	80
4	5	0.20	0.40	0.04	20
5	6	0.10	0.30	0.03	40

3. A one line diagram of the system is shown in the figure below. The system contains seven buses. The bus data, branch data and generator data are given below. The system data are prepared and the power flows are solved by Newton–Raphson method or fast decoupled method using **Power World Simulator software**.



Generator ratings

G_1: 100 MVA, 13.8 kV, $X'' = 0.12$, $X_2 = 0.14$, $X_0 = 0.05$ p.u.
G_2: 100 MVA, 13.8 kV, $X'' = 0.12$, $X_2 = 0.14$, $X_0 = 0.05$ p.u.

Generator neutrals are solidly grounded.

Transformer ratings

T_1: 100 MVA, 13.8 kVΔ/230 kV Y, $X = 0.1$ p.u.
T_2: 200 MVA, 15 kVΔ/230 kV Y, $X = 0.1$ p.u.

Generator neutrals are solidly grounded

Transmission line ratings

All lines: 230 kV, $Z_1 = 0.08 + j0.5$ Ω/km, $Z_0 = 0.2 + j1.5$ Ω/km, $y_1 = j3.3 \times 10^{-6}$ s/km
$L_1 = 15$ km, $L_2 = 25$ km, $L_3 = 40$ km, $L_4 = 15$ km, $L_5 = 50$ km

Power flow data

Bus 1: Swing bus, $V_1 = 13.8$ kV
Buses 2, 3, 4, 5, and 6: Load buses
Bus 7: Voltage control bus, $V_7 = 15$ kV, $P_{G7} = 180$ MW, -87 MVAR $< Q_{G7} < +87$ MVAR

System base quantities

$S_{\text{base}} = 100$ MVA, three phase, $V_{\text{base}} = 13.8$ kV in the zone of G_1